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Contents

Problems for nonlinear parabolic and elliptic-parabolic equations with variable exponents of nonlinearity in unbounded domains

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Let $n \in \mathbb{N}$ and $T > 0$ be arbitrary fixed numbers, Ω be an unbounded domain in $\mathbb{R}^n$, $\partial\Omega$ be the boundary of $\partial\Omega$ which is piecewise smooth surface, and $\nu = (\nu_1, ..., \nu_n)$ be the unit outward pointing normal vector on $\partial\Omega$. Suppose that $\partial\Omega = \Gamma_0 \cup \Gamma_1$, where Γ_0 is a closure of the open set in $\partial\Omega$ (in particular, Γ_0 can be empty set or coincide with $\partial\Omega$), $\Gamma_1 := \partial\Omega \setminus \Gamma_0$, $Q := \Omega \times (0, T)$, $\Sigma_0 := \Gamma_0 \times (0, T)$, and $\Sigma_1 := \Gamma_1 \times (0, T)$. Let $Bd(\Omega)$ be a set of all bounded subdomaines of domain Ω.

Suppose that

(B) *$b : \Omega \to \mathbb{R}$ is a measurable function, $b(x) \geq 0$ for almost every (a.e.) $x \in \Omega$, and $\operatorname{ess\,sup}_{x\in\Omega'} b(x) < \infty$ for every $\Omega' \in Bd(\Omega)$; there exists an open set $\Omega_0 \subset \Omega$ such that $b(x) > 0$ for a.e. $x \in \Omega_0$, and $b(x) = 0$ for a.e. $x \in \Omega \setminus \Omega_0$.*

We consider the problem: *to find the function $u : \overline{Q} \to \mathbb{R}$, which satisfies (in some sense) the equation*

$$\frac{\partial}{\partial t}(b(x)u) - \sum_{i=1}^{n} \frac{d}{dx_i} a_i(x, t, u, \nabla u) + a_0(x, t, u, \nabla u) = f_0(x, t) - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f_i(x, t), \quad (x, t) \in Q, \tag{1}$$

the boundary conditions

$$u\Big|_{\Sigma_0} = 0, \qquad \frac{\partial u}{\partial \nu_a}\Big|_{\Sigma_1} = 0, \tag{2}$$

and the initial condition

$$u(x, 0) = u_0(x)\,, \quad x \in \Omega_0, \tag{3}$$

where $a_j : Q \times \mathbb{R}^{1+n} \to \mathbb{R}$ $(j = \overline{0, n})$, $f_i : Q \to \mathbb{R}$ $(i = \overline{0, n})$, $u_0 : \Omega \to \mathbb{R}$ are given real-valued functions, $\frac{\partial u}{\partial \nu_a}(x, t) := \sum_{i=1}^{n} a_i(x, t, u, \nabla u)\, \nu_i(x)$, $(x, t) \in \Sigma_1$, is the exterior conormal derivative on $\partial\Omega$.

Assume that the spatial part of the differential expression in the left-hand side of (1) is elliptic, i.e., equation (1) is parabolic on $\Omega_0 \times (0, T)$ and is elliptic on $(\Omega \setminus \Omega_0) \times (0, T)$. In such way it is called the *elliptic-parabolic equation*.

A typical example of the equations of type (1), that are studying here, is

$$(b(x)u)_t - \sum_{i=1}^{n}\Big(\widehat{a}_i(x,t)|u_{x_i}|^{p_i(x)-2}u_{x_i}\Big)_{x_i} + \widehat{a}_0(x,t)|u|^{p_0(x)-2}u = f(x,t), \qquad (4)$$

where $\widehat{a}_i$, $i=\overline{0,n}$, are measurable, bounded, positive and separated from zero functions, $p_i > 1$, $i=\overline{0,n}$, are functions such that for all $i \in \{0,1,\dots,n\}$ the function $p_i : \Omega \to \mathbb{R}$ is measurable, $1 < \operatorname*{ess\,inf}_{x\in\Omega'} p_i(x) \le \operatorname*{ess\,sup}_{x\in\Omega'} p_i(x) < +\infty$ for each $\Omega' \Subset Bd(\Omega)$. Note that functions $p_i > 1$, $i=\overline{0,n}$, are called *exponents of the nonlinearity.*

As is well known, in order to guarantee the uniqueness of the solutions of the initial-boundary value problems for the linear and the some nonlinear parabolic equations in unbounded domains (such problems can be described in form (1)-(3)), we need to impose certain restrictions on their growth to infinity at $|x| \to +\infty$, for example, to require limitations of solutions or their belonging to some functional spaces. However, there are nonlinear parabolic equations, for which initial-boundary value problem is uniquely solvable without any conditions at infinity.

In recent decades, the nonlinear differential equations with the variable exponents of the nonlinearity are very active studied because such equations appear in mathematical modeling of the different physicals processes. In particular, these equations describe streams of the electroreological substances, recovering of the images, electric current in the conductor under the influence of the temperature field change.

In this topic we consider weak solutions of problems type (1)-(3) from the generalized Lebesgue and Sobolev spaces, and we prove uniquely solvability of the problems either with conditions at infinity or without it.

Stress state of isotropic plate weakened by two elliptical holes and crack

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In our time thin-walled structures of various configurations are widely used in construction, engineering, aircraft, rocket building and other branches of modern technology. In order to ensure the safe exploitation of such structures it is very important to investigate the tense state around cracks, openings and other stress concentrators.

This question was investigated earlier, but by other methods. Work [2] is devoted to the investigation of the stress state by the method of complex potentials. In works of

Dovbnya K. M. and her students different plates and shells of various configurations have been investigated. For example, [1] is devoted to non-through cracks and a circular hole. The problem in this work has been solved by the method of boundary integral equations.

Consider a thin isotropic plate of constant thickness h, weakened by two identical elliptical holes with major and minor axis $2a$ and $2b$ respectively, and a through crack with length of $2l$, located at the center of the plate between the holes. The plate is under the action of a symmetrical stretch along the axis Oy.

Assume that during the deformation of the plate the contours of the holes and the crack are free from loading, and the banks of the crack do not contact each other. In this case, the boundary conditions on the contours L_m ($m = 1, 2, 3$) will look like the following:

$$T_{n_m} = 0;\ S_{n\tau_m} = 0;\ M_{n_m} = 0;\ Q_{n_m} = 0,$$

where T_{n_m}, $S_{n\tau_m}$ are membrane efforts, M_{n_m} is bending moment, Q_{n_m} is generalized cross-cutting force.

Stress state in the plate due to the linearity of the problem is represented as the sum of the stress state in the solid plate, which is considered to be known, and the desired additional (perturbed) stress state caused by the presence of holes and a crack.

Assume the distance between holes, the crack and the outer contour is large in comparison with their size, and the perturbed stress state practically does not reach the outer contour of the plate. This allows instead of zero boundary conditions on the outer contour to set simplified conditions for the disappearance of perturbed stressed state with unlimited distance from L_m, and the area occupied by the plate is considered to be infinite.

To ensure the uniqueness of the solution at the ends of section L2, an additional condition

$$[v]_{L_2}|_{s=\pm 1_2} = 0$$

must hold.

One of the most effective methods for solving such problems is the method of boundary integral equations. Using the theory of generalized functions and the two-dimensional Fourier integral transform, the problem is reduced to a system of five singular integral equations with features of the Cauchy and Hilbert type.

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Kinetic Equations of Active Soft Matter

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We review a new approach to the description of the collective behavior of complex systems of mathematical biology within the framework of the evolution of observables. This representation of the kinetic evolution seems, in fact, the direct mathematically fully consistent formulation modeling kinetic evolution of biological systems since the notion of the state is more subtle and it is an implicit characteristic of populations of living creatures.

One of the advantages of the developed approach is the opportunity to construct kinetic equations in scaling limits, involving initial correlations, in particular, that can characterize the condensed states of soft matter. We note also that such approach is also related to the problem of a rigorous derivation of the non-Markovian kinetic-type equations from underlying many-cell dynamics which make it possible to describe the memory effects of the kinetic evolution of cells.

Using suggested approach, we establish a mean field asymptotic behavior of the hierarchy of evolution equations for marginal observables of a large system of interacting stochastic processes of collisional kinetic theory, modeling the microscopic evolution of active soft condensed matter.

Furthermore, we established that for initial states specified by means of a one-particle distribution function and correlation functions the evolution of additive-type marginal observables is equivalent to a solution of the Vlasov-type kinetic equation with initial correlations, and a mean field asymptotic behavior of non-additive type marginal observables is equivalent to the sequence of explicitly defined correlation functions which describe the propagation of initial correlations of active soft condensed matter.

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Asymptotic analysis of a spectral Robin problem in a thick junction with the branched fractal structure

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The asymptotic behavior of the eigenvalues and eigenfunctions of a mixed boundary-value problem for the Laplace equation in a thick junction with the branched fractal structure and the perturbed Robin boundary conditions on the boundaries of the branches is studied. The Hausdorff convergence of the spectrum is proved, the leading terms of asymptotics are constructed and the corresponding asymptotic estimates are justified both for the eigenvalues and eigenfunctions

Stochastic Kinetic Mean Field Approach versus Classical Nucleation Theory

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Nucleation stage of the first-order phase transformations in alloys is crucial for prediction of mechanical, electrical, magnetic properties of multiphase materials. During last decades the new experimental possibilities have been developed enabling direct observation of nuclei formation during aging and solid-state reactions. Yet many details of the nucleation stage still remain the mystery.

Two most widespread methods of the nucleation kinetics investigation are Fokker-Plank approach and Monte Carlo simulation. Fokker-Plank approach seems a good solution but it contains a number of not very well determined parameters and not very well proved phenomenological assumptions. Monte Carlo is more direct and atomistic, but the level of fluctuations in this method is so high that it is very difficult to distinguish the structures in small volumes.

Recently, our group, jointly with the group of Debrecen University, developed the new method called Stochastic Kinetic Mean Field (SKMF) [1-3]. This method combines George Martin‘s mean field atomistic approach with the noise of local atomic fluxes. SKMF approach is inherently nonlinear and therefore applicable to the early stages of solid-state reactions under sharp concentration gradient. In this approach the probability of atomic exchanges is proportional to difference of exponents of chemical potentials, instead of difference of just chemical potentials. Moreover, noise is introduced directly into atomic jumps quantity, instead of noise of composition.

Stochastic Kinetic Mean Field modeling on nucleation in supersaturated solution demonstrates the validity of Classical Nucleation Theory. The nucleation process consists of two maim steps: at first, the embryo of new phase appears with almost optimal composition and then this embryo increases its size at almost constant composition.Logarithm of nucleation time is inversely proportional to the squared supersaturation. Logarithm of nucleation time is a linear function of the inverse squared noise amplitude.

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Asymptotic solutions of soliton type to the Korteweg-de Vries equation with singular perturbation and variable coefficients

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The Korteweg-de Vries equation

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1}$$

describes a lot of interesting phenomena and processes in liquids, solid body, plasma as well as in optic, nuclear, biological and telecommunication systems. In particular, it simulates propagation of solitary waves in homogeneous medium with nonlinear dispersion.

In the case of the medium with variable characteristics and small dispersion it is necessary to consider the Korteweg-de Vries equation with singular perturbation and variable coefficients that is studied at the moment. For example, using nonlinear [1] WKB method V. P. Maslov, Yu. S. Dobrokhotov and G. A. Omel'yanov have constructed asymptotic solutions to the following equation [2]

$$u_t + (\rho_1 + 3\rho_2 u)u_x + \varepsilon^2 \rho_3 u_{xxx} + \rho_4 u = 0, \quad x \in \mathbf{R}, \quad \in [0;T], \tag{2}$$

where $\rho_1 = \sqrt{gH(x)}$, $\rho_2 = \sqrt{gH^{-1}(x)}/2$, $\rho_3 = \sqrt{gH^5(x)}/6$, $\rho_4 = \rho_{1x}/2$, $H(x) > 0$ is depth of non-perturbed liquid, g is acceleration of gravity, ε is a small parameter characterized value of dispersion.

The constructed asymptotic solutions to equation (2) have been called *soliton-like* solutions [2] since accordingly to their structure, the approximate solutions are asymptotically close to soliton solutions. This property allows to construct approximate wave solutions to differential equations of integrable type with perturbations because their solutions are some deformations of corresponding soliton solutions to certain equations. Thus, the concept of soliton-like solutions of integrable type equations with variable coefficients and small perturbation was proposed.

We study the singularly perturbed Korteweg-de Vries equation with variable coefficients

$$\varepsilon^n u_{xxx} = a(x,t,\varepsilon)u_t + b(x,t,\varepsilon)uu_x, \tag{3}$$

where n is natural, the functions $a(x,t,\varepsilon)$, $b(x,t,\varepsilon)$ are represented as asymptotic series

$$a(x,t,\varepsilon) = \sum_{j=0}^{\infty} a_j(x,t)\varepsilon^j, \quad b(x,t,\varepsilon) = \sum_{j=0}^{\infty} b_j(x,t)\varepsilon^j,$$

the functions $a_j(x,t)$, $b_j(x,t)$ are infinitely differentiable with respect to variables $(x,t) \in \mathbf{R} \times [0;T]$ for all $j \geq 0$ and $\varepsilon > 0$ is a small parameter.

In [1, 4, 5], a technique was developed for constructing asymptotic soliton-like solutions of equation (3) through the non-linear WKB method and its justification was given. In particular, it turned out that structure of asymptotic solutions essentially depends on degree of a small parameter in the equation (3) and asymptotic one phase soliton-like solution to equation (3) as $n = 2$ was found to be written as follows

$$u(x,t,\varepsilon) = U_N(x,t,\varepsilon) + V_N(x,t,\tau,\varepsilon) + O\left(\varepsilon^{N+1}\right).$$

Here $U_N(x,t,\varepsilon) = \sum\limits_{j=0}^{N} \varepsilon^j u_j(x,t)$ is regular part of the asymptotic solution $u(x,t,\varepsilon)$, $V_N(x,t,\tau,\varepsilon) = \sum\limits_{j=0}^{N} \varepsilon^j V_j(x,t,\tau)$ is its singular part and $\tau = (x - \varphi(t))/\varepsilon$ is a phase variable. Function $x - \varphi(t)$ is called a phase of the one phase soliton-like solution and $\varphi(t) \in C^\infty([0;T])$ defines the discontinuity curve [2]: $\Gamma = \{(x,t) \in \mathbf{R} \times [0;T] : x = \varphi(t)\}$. The terms $V_0(x,t,\tau) \in G_1^0$, $V_j(x,t,\tau) \in G_1$, $j = \overline{1,N}$, where G_1^0, G_1 are certain functional spaces.

It should be noted that the regular part is background function while the singular part reflects soliton properties of the constructed asymptotic solution. The last circumstance is taken into account when determining the functional spaces to which the terms of the singular part should belong. The spaces G_1^0, G_1 are defined as the following [2]: $G_1 = G_1(\mathbf{R} \times [0;T] \times \mathbf{R})$ is a linear space of infinitely differentiable functions $f = f(x,t,\tau)$, $(x,t,\tau) \in \mathbf{R} \times [0;T] \times \mathbf{R}$, such that for any non-negative integers n, p, q, r uniformly with respect to (x,t) on any compact set $K \subset \mathbf{R} \times [0;T]$ the conditions are fulfilled:

1^0. the relation

$$\lim_{\tau \to +\infty} \tau^n \frac{\partial^p}{\partial x^p} \frac{\partial^q}{\partial t^q} \frac{\partial^r}{\partial \tau^r} f(x,t,\tau) = 0, \quad (x,t) \in K,$$

is satisfied;

2^0. there exists such an infinitely differentiable function $f^-(x,t)$ that

$$\lim_{\tau \to -\infty} \tau^n \frac{\partial^p}{\partial x^p} \frac{\partial^q}{\partial t^q} \frac{\partial^r}{\partial \tau^r} \big(f(x,t,\tau) - f^-(x,t) \big) = 0, \quad (x,t) \in K.$$

The space $G_1^0 = G_1^0(\mathbf{R} \times [0;T] \times \mathbf{R})$ is a subspace of G_1 and contains functions $f = f(x,t,\tau)$ that satisfy the following condition:

$$\lim_{\tau \to -\infty} f(x,t,\tau) = 0$$

uniformly with respect to variables (x,t) on any compact $K \subset \mathbf{R} \times [0;T]$.

We consider two types of the constructed asymptotic soliton-like solutions depending on properties of the terms of its singular part. The first type contains solutions all singular terms of which tend to zero as phase argument infinitely grows in both positive and negative direction. Their singular parts are represented as functions belonging to the space of quickly decreasing functions with respect to phase variable. These solutions are called asymptotic solutions of soliton type.

For the solutions of the other type the singular terms don't have this property. It means that these functions are not quickly decreasing functions with respect to phase variable. More exactly, they may have non-zero asymptotic as phase variable $\tau \to -\infty$.

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On $\mathfrak{B}_{1,s}$ classes of De Giorgi-Ladyzhenskaya-Ural'tseva and their applications to elliptic and parabolic equations with nonstandard growth

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We define elliptic and parabolic $\mathfrak{B}_{1,s}$ classes, which generalize the well known $\mathfrak{B}_p$ classes of De Giorgi-Ladyzhenskaya-Ural'tseva [1], [2]. We show that solutions of the equations

$$\operatorname{div}(|\nabla u|^{p(x)-2}\,\nabla u)=0, \qquad u_t-\operatorname{div}(|\nabla u|^{p(x,t)-2}\,\nabla u)=0,$$

$$\operatorname{div}\left(g(|\nabla u|)\frac{\nabla u}{|\nabla u|}\right)=0, \qquad u_t-\operatorname{div}\left(g(|\nabla u|)\frac{\nabla u}{|\nabla u|}\right)=0,$$

$$\left(\frac{w}{v}\right)^{p-1}\leqslant\frac{g(w)}{g(v)}\leqslant\left(\frac{w}{v}\right)^{q-1}, \quad w\geqslant v>0, \quad p<q,$$

$$\operatorname{div}(|\nabla u|^{p-2}\,\nabla u+a(x)|\nabla u|^{q-2}\,\nabla u)=0, \quad a(x)\geqslant 0,$$

$$u_t-\operatorname{div}(|\nabla u|^{p-2}\,\nabla u+a(x,t)|\nabla u|^{q-2}\,\nabla u)=0, \quad a(x,t)\geqslant 0,$$

$$(-1)^m\sum_{|\alpha|=m}D^\alpha\left[\left(\sum_{|\beta|=m}|D^\beta u|^2\right)^{(p-2)/2}D^\alpha u\right]-\operatorname{div}(|\nabla u|^{q-2}\,\nabla u)=0, \quad q>mp,$$

$$u_t-(-1)^m\sum_{|\alpha|=m}D^\alpha\left[\left(\sum_{|\beta|=m}|D^\beta u|^2\right)^{(p-2)/2}D^\alpha u\right]-\operatorname{div}(|\nabla u|^{q-2}\,\nabla u)=0, \quad q>mp,$$

belong to the correspondent $\mathfrak{B}_{1,s}$ classes and hence are locally Hölder continuous.

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Differential-delay equations and mobility of solitons in condensed matter physics

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Mobility of topological solitons in discrete media is generally suppressed. The reason for this is the resonance between the topological soliton and the linear spectrum of the system. More precisely, for any topological soliton velocity a plane wave with the same phase velocity can be excited. This does not happen in the continuous systems, where solitons and phonons occupy different velocity sectors.

We are interested in the existence of the travelling wave solutions of the discrete generalized nonlinear Klein-Gordon equation

$$\ddot{u}_n - U'(u_{n+1} - u_n) + U'(u_n - u_{n-1}) + V'(u_n) = 0, \quad n \in \mathbb{Z}, \tag{1}$$

where the on-site potential $V(u)$ has at least two degenerate minima and $U(r)$ is the interaction potential that in the simplest case can be harmonic: $U(r) = \kappa r^2/2$. If we are looking for the travelling wave solutions of the type $u_n(t) = u(n - vt) \equiv u(z)$ the equation of motion (1) becomes the differential-delay ODE:

$$v^2 u''(z) - U'[u(z+1) - u(z)] + U'[u(z) - u(z-1)] + V'[u(z)] = 0\ .$$

In this research we demonstrate the existence of a selected set of velocities for which the travelling wave solutions have exponentially decaying tails. These moving solitons that exist for some selected values of velocity are discrete *embedded* solitons. Embedded solitons are solitons that exist despite the resonance with the linear spectrum of the underlying system. Moving lattice solitons fit perfectly into this definition because they exist despite the fact that for any soliton velocity a linear wave with the same phase velocity can be excited.

We discuss existence of moving embedded solitons in arrays of Josephson junctions [1, 2] and crystals of *mica muscovite* [3] as well as numerical methods for finding these solutions.

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SHORT COMMUNICATIONS

Stability of viscoelastic wave equation with strong delay

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Consider a viscoelastic body occupying in its reference configuration, in which renders the body is free of any stresses, a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$, $d \in \mathbb{N}$. We further consider an initial-boundary value problem for a viscoelastic wave equation subject to a strong time-localized delay in the Kelvin & Voigt-type material law. Imposing mixed homogeneous Dirichlet–Neumann boundary conditions on y and usual initial conditions, the system of partial delay differential equations reads as ([1])

$$\begin{gathered}\partial_{tt} y(t,x) - c_1 \triangle y(t,x) - c_2 \triangle y(t-\tau,x) \\ -d_1 \partial_t \triangle y(t,x) - d_2 \partial_t \triangle y(t-\tau,x) = 0 \text{ for } t>0,\ x\in\Omega, \end{gathered} \tag{1}$$

$$y(t,x) = 0 \text{ for } t>0,\ x\in\Gamma_0, \quad \frac{\partial y(t,x)}{\partial \nu} = 0 \text{ for } t>0,\ x\in\Gamma_1, \tag{2}$$

$$y(0+,x) = y^0, \quad \partial_t y(0+,x) = y^1 \text{ for } x\in\Omega, \quad y(t,x) = \varphi(t,x) \text{ for } (t,x)\in[-\tau,0]\times\Omega, \tag{3}$$

where $\nu\colon \Gamma \to \mathbb{R}^3$ stands for the outer unit normal vector to the boundary Γ and $\frac{\partial u}{\partial \nu}$ is the normal derivative, $\tau > 0$ is a delay time, and c_1, c_2, d_1, d_2 are positive real numbers.

Following [1] and introducing the 'history variable'

$$z(s,t,x) = y(t-\tau s, x), \quad s\in(0,1), \quad t>0, \tag{4}$$

we define the extended phase space

$$\mathcal{H} = H^1_{\Gamma_0}(\Omega) \times L^2(\Omega) \times L^2_\tau\big(0,1; H^1_{\Gamma_0}(\Omega)\big) \times L^2_\tau\big(0,1; H^1_{\Gamma_0}(\Omega)\big)$$

equipped with the inner product

$$\langle U, V\rangle_{\mathcal{H}} = \int_G \left(c_1 \nabla u_1 \cdot \nabla v_1 + u_2 v_2 + \tau d_1 \int_0^1 \nabla u_3 \cdot \nabla v_3 \,\mathrm{d}s + \tau d_2 \int_0^1 \nabla u_4 \cdot \nabla v_4 \,\mathrm{d}s \right) \mathrm{d}x, \tag{5}$$

for $U = (u_1,u_2,u_3,u_4)^T, V = (v_1,v_2,v_3,v_4)^T \in \mathcal{H}$, where $H^1_{\Gamma_0}(\Omega) = \big\{u\in H^1(\Omega) \,|\, u|_{\Gamma_0} = 0\big\}$. It is easy to verify that the topology induced by the inner product is equivalent with the standard product topology on $\mathcal{H}$.

Consider the linear operator $\mathcal{A}\colon D(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ defined via

$$\mathcal{A}V = \begin{pmatrix} v_2 \\ c_1\triangle v_1 + c_2 \triangle v_3|_{s=1} + d_1 \triangle v_2 + d_2 \triangle v_4|_{s=1} \\ -\tau^{-1}\partial_s v_3 \\ -\tau^{-1}\partial_s v_4 \end{pmatrix} \quad \text{for} \quad V = (v_1,v_2,v_3,v_4)^T, \tag{6}$$

with an appropriate domain $D(\mathcal{A})$.

Then Equations (1)–(3) then can be transformed to an abstract Cauchy problem on the extented phase space $\mathcal{H}$

$$\dot{V}(t) = \mathcal{A}V(t) \quad \text{for} \quad t > 0, \quad V(0) = V^0 \tag{7}$$

with $V = (v_1, v_2, v_3, v_4)^T$ and $V^0 := (y^0, y^1, \varphi^0, \varphi^1)^T$, where $\varphi^0 := \varphi\big(-\frac{\cdot-\tau}{\tau}\big)$, $\varphi^1 := (\partial_t\varphi)\big(-\frac{\cdot-\tau}{\tau}\big)$.

Assumption 1. *Suppose the coefficients $c_1, c_2, d_1, d_2 > 0$ satisfy the condition*

$$2d_1 \geq d_2 + \sqrt{d_2^2 + 2c_2^2}.$$

Straightforward checking of density, disipativity, surjectivity of corresponding operators and application of Lumer-Phillips' Theorem [2] characterize $\mathcal{A}$ as an m-dissipative operator on $\mathcal{H}$.

Now, by virtue of [3], it follows the abstract formulation (7) of Equations (1)–(3) is Hadamard well-posed.

Theorem. *For any $V^0 \in \mathcal{H}$ and $F \in L^2_{\text{loc}}\big(0, \infty; \mathcal{H}\big)$, there exists a unique mild solution $V \in C^0\big([0, \infty), \mathcal{H}\big)$ to Equation (7). Moreover, if $V^0 \in D(\mathcal{A})$ and*

$$F \in C^1\big([0, \infty), \mathcal{H}\big) \cup C^0\big([0, \infty), D(\mathcal{A})\big),$$

the mild solution is classical:

$$V \in C^1\big([0, \infty), \mathcal{H}\big) \cap C^0\big([0, \infty), D(\mathcal{A})\big).$$

Now we consider the problem of exponential stability. The 'natural' energy reads as

$$\mathcal{E}(t) = \frac{1}{2}\|\partial_t y\|^2 + \frac{c_1}{2}\|\nabla y\|^2 + \frac{\tau d_1}{2}\int_0^1 \|\nabla z(s, \cdot)\|^2 \mathrm{d}s + \frac{\tau d_2}{2}\int_0^1 \|\partial_t \nabla z(s, \cdot)\|^2 \mathrm{d}s \tag{8}$$

In addition to Assumption 1, suppose:

Assumption 2. *Suppose the coefficients c_1, c_2, d_1, d_2 satisfy:*

1. $c_1 > 6c_2$,

2. $d_1^2 \geq \max\left\{\frac{9c_1^2 d_2^2}{c_1^2 - 9c_2^2}, \frac{18c_1 c_2^2 c_p}{c_1^2 - 36c_2^2}\right\}$.

Then we can formulate the theorem on exponential stability of the system.

Theorem. *Let $V^0 \in \mathcal{H}$. Under Assumptions 1, 2, there exist constants $\tilde{\alpha}, C > 0$ such that*

$$\mathcal{E}(t) \leq Ce^{-\tilde{\alpha}t}\mathcal{E}(0).$$

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Application of Hayman's theorem to linear directional differential equations having analytic solutions in the unit ball and boundedness of L-index in direction

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Let $\mathbf{0} = (0, \ldots, 0)$, $\mathbf{b} = (b_1, \ldots, b_n) \in \mathbb{C}^n \setminus \{\mathbf{0}\}$ be a given direction, $\mathbb{R}_+ = (0, +\infty)$, $\mathbb{B}^n = \{z \in \mathbb{C}^n : |z| < 1\}$, $L: \mathbb{B}^n \to \mathbb{R}_+$ be a continuous function such that for all $z \in \mathbb{B}^n$ one has

$$L(z) > \frac{\beta|\mathbf{b}|}{1-|z|}, \ \beta = \text{const} > 1. \tag{1}$$

Analytic function $F : \mathbb{B}^n \to \mathbb{C}$ is called [1] a function of *bounded L-index in a direction* $\mathbf{b}$ if there exists $m_0 \in \mathbb{Z}_+$ such that for every $m \in \mathbb{Z}_+$ and every $z \in \mathbb{B}^n$ the following inequality is valid

$$\frac{|\partial_{\mathbf{b}}^m F(z)|}{m! L^m(z)} \le \max\left\{\frac{|\partial_{\mathbf{b}}^k F(z)|}{k! L^k(z)} : 0 \le k \le m_0\right\},$$

where $\partial_{\mathbf{b}}^0 F(z) = F(z), \partial_{\mathbf{b}} F(z) = \sum\limits_{j=1}^n \frac{\partial F(z)}{\partial z_j} b_j$, $\partial_{\mathbf{b}}^k F(z) = \partial_{\mathbf{b}}\Big(\partial_{\mathbf{b}}^{k-1} F(z)\Big)$, $k \ge 2$. Let $\mathbb{D} = \{t \in \mathbb{C} : |t| < 1\}$, $L : \mathbb{B}^n \to \mathbb{R}_+$ be a continuous function. For $z \in \mathbb{B}^n$ we denote $D_z = \{t \in \mathbb{C} : |t| \le \frac{1-|z|}{|\mathbf{b}|}\}$,

$$\lambda_{\mathbf{b}}(\eta) = \sup_{z\in\mathbb{B}^n} \sup_{t_1,t_2\in D_z} \left\{\frac{L(z+t_1\mathbf{b})}{L(z+t_2\mathbf{b})} : |t_1 - t_2| \le \frac{\eta}{\min\{L(z+t_1\mathbf{b}), L(z+t_2\mathbf{b})\}}\right\}.$$

The notation $Q_{\mathbf{b}}(\mathbb{B}^n)$ stands for a class of positive continuous functions $L : \mathbb{B}^n \to \mathbb{R}_+$, satisfying (1) and $(\forall \eta \in [0, \beta]) : \lambda_{\mathbf{b}}(\eta) < +\infty$.

We denote $a^+ = \max\{a, 0\}$. Set $u(r) = u(z^0, \theta, r) = L(z^0 + re^{i\theta}\mathbf{b})$. Let $W_{\mathbf{b}}(\mathbb{B}^n)$ be a class of positive continuous function $L : \mathbb{B}^n \to \mathbb{R}_+$ satisfying (1) and the following conditions:

1) for every $z^0 \in \mathbb{B}^n$ and every $\theta \in [0, 2\pi]$ the function $u(r, z^0, \theta)$ be a continuously differentiable function of real variable $r \in [0, r_0)$, where $r_0 = \min\{s \in \mathbb{R}_+ : \ |z^0 + se^{i\theta}\mathbf{b}| = 1\}$;

2) for every $z^0 \in \mathbb{B}^n$, $\theta \in [0, 2\pi]$ one has $\left(\frac{d}{ds}\frac{1}{L(z^0+sre^{i\theta}\mathbf{b})}\big|_{s=1}\right)^+ \to 0$ as $|z^0 + re^{i\theta}\mathbf{b}| \to 1$, i.e. $r \to r_0$.

We formulate proposition containing growth estimates of analytic solutions of the partial differential equation

$$g_0(z)\partial_{\mathbf{b}}^p F(z) + g_1(z)\frac{\partial^{p-1}F(z)}{\partial\mathbf{b}^{p-1}} + \cdots + g_p(z)F(z) = h(z). \tag{2}$$

Let us denote $QW_{\mathbf{b}}(\mathbb{B}^n) = Q_{\mathbf{b}}(\mathbb{B}^n) \cap W_{\mathbf{b}}(\mathbb{B}^n)$.

Theorem. *Let $L \in QW_{\mathbf{b}}(\mathbb{B}^n)$, functions g_0, g_1, .., g_p, and h be analytic in the unit ball and there exists $R \in [0, 1)$ such that for all $z \in \mathbb{B}^n$, $|z| \geq R$, the following conditions hold*

1) $|g_j(z)| \leq m_j L^j(z)|g_0(z)|$ *for* $1 \leq j \leq p$;

2) $|\partial_{\mathbf{b}} g_j(z)| < M_j \cdot L^{j+1}(z)|g_0(z)|$ *for* $0 \leq j \leq p$;

3) $|\partial_{\mathbf{b}} h(z)| \leq M \cdot L(z) \cdot |h(z)|$,

where m_j and M are nonnegative constants and M_j are positive constants. If an analytic function $F : \mathbb{B}^n \to \mathbb{C}$ satisfies equation (2) and $\forall z \in \mathbb{B}^n$, $|z| < R$, $F(z + t\mathbf{b}) \not\equiv 0$ then F has bounded L-index in the direction $\mathbf{b}$ and for all $z^0 \in \mathbb{B}^n$, $\theta \in [0, 2\pi]$

$$\varlimsup_{|z^0+re^{i\theta}\mathbf{b}|\to 1} \frac{\ln|F(z^0 + e^{i\theta}r\mathbf{b})|}{\int_0^r L(z^0 + te^{i\theta}\mathbf{b})dt} \leq \max\{1, C\}, \tag{3}$$

where $C = \sum_{j=1}^{p} M_j + (M+1)\sum_{j=1}^{p} m_j + M$.

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The nonlocal boundary value problem with perturbations of mixed boundary conditions for differential equation with constant coefficients

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In domain $G := \{x := (x_1, x_2) \in \mathbb{R}^2 : 0 < x_1,\ x_2 < 1\}$, we consider the multipoint problem

$$L(-D_1^2, -D_2^2)u := \sum_{q=0}^{n} a_q D_1^{2q} D_2^{2n-2q} u = f(x),\ x \in G, \tag{1}$$

$$\ell_{s,1}u := D_1^{2s-2}u|_{x_1=0} + D_1^{2s-2}u|_{x_1=1} + \ell_{s,1}^1 u = 0, \tag{2}$$

$$\ell_{n+s,1}u := D_1^{2s-2}u|_{x_1=0} - D_1^{2s-2}u|_{x_1=1} = 0, \tag{3}$$

$$\ell_{s,2}u := D_2^{2s-2}u|_{x_1=0} + D_2^{2s-2}u|_{x_1=1} + \ell_{s,2}^0 u = 0, \tag{4}$$

$$\ell_{n+s,2}u := D_2^{2s-1}u|_{x_2=0} + D_2^{2s-1}u|_{x_2=1} = 0, \tag{5}$$

where

$$\ell_{s,j}^0 u := \sum_{q=0}^{k_{s,j}} \sum_{r=0}^{n_j} b_{s,q,r,j} D_j^q u(x_1, x_2)|_{x_j = x_{j,r}},\ s = 1, ..., n, \tag{6}$$

$$0 = x_{j,1} < x_{j,2}, ..., x_{j,n_j} \le 1,\ b_{s,q,r,j} \in \mathbb{R},$$

$$q = 0, 1, ..., k_{s,j},\ k_{s,j} < 2n,\ r = 0, 1, ..., n_j,\ s = 1, \dots, n,\ j = 1, 2.$$

Assumption P_1 : $b_{s,q,r,j} = -(-1)^q b_{s,q,n_j-r,j},\ x_{j,r} = 1 - x_{j,n_j-r},\ r = 0, 1, ..., n_j,\ s = 1, \dots, n,\ j = 1, 2.$

Assumption P_2 : $k_{s,j} \le 2s - 2,\ s = 1, ..., n,\ j = 1, 2.$

Assumption P_3 : *for any real numbers* $\mu_1,\ \mu_2$ *the positive number* C_1 *exists, that the inequality* $C_1|\mu|^n \le |L(\mu_1, \mu_2)|,\ \mu := (\mu_1, \mu_2),\ |\mu|^2 := |\mu_1|^2 + |\mu_2|^2$, *holds.*

Assumption P_4 : $a_0 a_n \ne 0$.

Let $L : L_2(G) \to L_2(G)$ be the operator of the problem (1)–(6) and

$Lu := L(-D_1^2, D_2^2)u,\ u \in D(L),\ D(L) := \{u \in W_2^{2n}(G) : \ell_{s,j}u = 0,\ s = 1, \dots, 2n,\ j = 1, 2\}.$

Let us formulate the main results of this work.

Theorem 1. Let Assumption P_1 holds. Therefore, for arbitrary $a_q \in \mathbb{R},\ q = 0, 1, ..., n,\ b_{s,q,r,j} \in \mathbb{C}$, the operator L has a set of eigenvalues

$$\sigma := \{\lambda_{k,m} := L(\mu_{1,k}, \mu_{2,m}),\ \mu_{1,k} = \pi^2 k^2,\ \mu_{2,m} = \pi^2 (2m-1)^2,\ k \in \mathbb{N},\ m \in \mathbb{N}\}, \tag{7}$$

and the system $V(L)$ of root functions, which is complete and minimal in the space $L_2(G)$.

Theorem 2. Let Assumptions P_1–P_3 hold and $a_0 a_n \neq 0$. Therefore, the operator L has the system $V(L)$ of root functions, which is the Riesz basis of the space $L_2(G)$.

Theorem 3. Let Assumptions P_1–P_3 hold and $a_0 a_n \neq 0$. Therefore, for arbitrary function $f \in L_2(G)$ the unique solution $u \in W_2^{2n}(G)$ of problem (1)–(6) exists.

Averaging method in multifrequency systems of ODE and PDE with delay and integral conditions

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We consider a system of differential equations of the form

$$\frac{da}{d\tau} = X(\tau, a_\Lambda, \varphi_\Theta), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y(\tau, a_\Lambda, \varphi_\Theta), \tag{1}$$

with the conditions

$$\sum_{\nu=1}^{r} b(t_\nu) a(t_\nu) = d_1, \quad 0 \leq t_1 < \ldots t_r \leq L, \tag{2}$$

$$\int_0^{\tau_1} \big[g_1(\tau, a_\Lambda(\tau))\varphi_\Theta(\tau) + f_1(\tau, a_\Lambda(\tau), \varphi_\Theta(\tau))\big] d\tau +$$

$$+ \int_{\tau_2}^{L} \big[g_2(\tau, a_\Lambda(\tau))\varphi_\Theta(\tau) + f_2(\tau, a_\Lambda(\tau), \varphi_\Theta(\tau))\big] d\tau = d_2. \tag{3}$$

Here $0 \leq \tau \leq L$, $(0, \varepsilon_0] \ni \varepsilon$ – a small parameter, $0 < \tau_1 < \tau_2 < L$, $a \in D \subset \mathbb{R}^n$, $\varphi \in \mathbb{T}^m$, $\Lambda = (\lambda_1, \ldots, \lambda_p)$, $\Theta = (\theta_1, \ldots, \theta_q)$, $\lambda_i, \theta_j \in (0,1)$, $x_{\lambda_i}(\tau) = x(\lambda_i \tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$.

The complexity of the research of the problem (1)–(3) is the existence of resonances. Resonance condition in point $\tau \in [0, L]$ is

$$\sum_{\nu=1}^{q} \theta_\nu (k_\nu, \omega(\theta_\nu \tau)) = 0, \quad k_\nu \in \mathbb{R}^m, \quad \|k\| \neq 0.$$

Multifrequency systems of ODE were considered in [1] by averaging method, and systems with delay of argument – in [2], [3] and others.

Averaging in system (1) and condition (3) is carried out on fast variables φ_θ on the torus T^m. The averaged problem takes the form

$$\frac{d\overline{a}}{d\tau} = X_0(\tau, \overline{a}_\Lambda), \quad \frac{d\overline{\varphi}}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y_0(\tau, \overline{a}_\Lambda), \tag{4}$$

$$\sum_{\nu=1}^{r} b(t_\nu)\overline{a}(t_\nu) = d_1, \tag{5}$$

$$\int_0^{\tau_1} \left[g_1(\tau, \overline{x}_\Lambda(\tau))\overline{\varphi}_\Theta(\tau) + f_1(\tau, \overline{x}_\Lambda(\tau))\right] d\tau +$$

$$+ \int_{\tau_2}^{L} \left[g_2(\tau, \overline{x}_\Lambda(\tau))\overline{\varphi}_\Theta(\tau) + f_2(\tau, \overline{x}_\Lambda(\tau))\right] d\tau = d_2. \tag{6}$$

$$\int_{\tau_1}^{\tau_2} \left[g(\tau, \overline{x}_\Lambda(\tau))\overline{\varphi}_\Theta(\tau) + f_0(\tau, \overline{x}_\Lambda(\tau))\right] d\tau = d_2. \tag{7}$$

The existance of solutions of averaged problem (4)–(6) and problem (1)–(3) was proved. Conditions, under which the assessment

$$\|x(\tau, \varepsilon) - \overline{x}(\tau)\| \leq c\varepsilon^\alpha, \quad 0 < \alpha \leq (mq)^{-1}$$

is true on $[0, \tau]$, were found.

The object of this paper is the Darboux problem [4] in the form

$$\frac{\partial^2 u}{\partial\tau\partial x} = f(\tau, x, u_{\Lambda\Theta}, a_\Lambda, \varphi_\Theta), (\tau, x) \in [0, L] \times [0, M], \tag{7}$$

$$u(\tau, 0) = \mu(\tau), \tau \in [0, L], u(0, x) = \xi(x), x \in [0, M], \tag{8}$$

where a and φ are the solutions of system of differential equations (1) with the conditions (2), (3).

The system of equations (7) averaged by the vector of fast variables $\varphi_\Theta \in T^{mq}$ takes the form

$$\frac{\partial^2 \overline{u}}{\partial\tau\partial x} = f_0(\tau, x, \overline{u}_{\Lambda\Theta}, \overline{a}_\Lambda),$$

$$\overline{u}(\tau, 0) = \mu(\tau), \tau \in [0, L], \overline{u}(0, x) = \xi(x), x \in [0, M], \tag{9}$$

Existence and uniqueness of solution of the averaged problem and the problem (1) – (3), (7) – (8) are proved and averaging method in resonance case is grounded. Conditions, under which the assessment

$$|u(\tau, x, \varepsilon) - \overline{u}(\tau, x)| + ||a(\tau, \varepsilon) - \overline{a}(\tau)|||| \leq c\varepsilon^\alpha, 0 < \alpha \leq (mq)^{-1}$$

for all $\varepsilon \in (0, \varepsilon_0]$, $\varepsilon_0 << 1$, $(\tau, x) \in [0, L] \times [0, M]$ is true, were found.

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Some problems of difference equations computer calculations

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The theory of difference schemes of numerical solution of differential equations is one of the main parts of modern computational mathematics. The solution of differential equations by analytical and iterative (numerical) methods is chosen for reasons of computational efficiency of the computation algorithm. But quit important is the issue of accuracy and stability at the stage of computer computing. Ignoring peculiarities of computer computing can lead to incorrect and paradoxical results, even in the case of mathematical examples that are quite simple.

Let's consider a well-known example [1], the analysis of which was performed by W. Kahan

$$x_n = 111 - \frac{1130}{x_{n-1}} + \frac{3000}{x_{n-1} \cdot x_{n-2}},$$

with initial conditions $x_0 = 2, x_1 = -4$.

Having constructed the appropriate characteristic equation, we can conclude that if this sequence coincides then it can coincide to one of three numbers: 5, 6 or 100. In this case, for certain initial indications, there may occur paradoxes in the calculations: a greater number of members of a sequence leads to an incorrect result. This happens due to the way of computer calculations organization and the limited amount of memory that is allocated to processing a real number. With given initial conditions $x_0 = 2, x_1 = -4$, the sequence x_n should coincide to the value of 6, but standard computer calculations will show a result that is close to 100. To demonstrate this fact it's enough to consider a simple program implemented using the programming language Python 3.x the code of which is given below

```
from decimal import Decimal
x0 = Decimal(2); x1 = Decimal(-4)
for i in range(50):
    xn = Decimal(111) - Decimal(1130)/x1 + Decimal(3000)/x0/x1
    x0 = x1; x1 = xn
    print(xn)
```

Note the fact that in the program implementation the Decimal class was used, which should provide more accurate results. It is worth to noting that in the range of values from x_2 to x_{25} we observe a downward sequence that goes to the expected value of 6. Starting from the value of x_{26} the sequence begin to increase until it is fixed at the value of 100. If you reduce the accuracy of calculations (for example, do not use the class Decimal) violation of the convergence process to the real limit begins earlier, already with value of x_{16}. In this work we research mathematical reasons of appearing of the incorrect results of calculations of this kind, formalizing and generalizing the approach proposed by W. Kahan.

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Singular perturbed problems of Orr-Zommerfeld type with nonstable turning point

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We considered a singular perturbed system of linear ordinary differential equations of the following type:

$$\varepsilon Y'(x,\varepsilon) - A(x,\varepsilon)Y(x,\varepsilon) = H(x), \tag{1}$$

when $\varepsilon \to 0$, $x \in [-l,l]$, $Y(x,\varepsilon) = Y_k(x,\varepsilon) = col(y_i(x,\varepsilon))$, $i = \overline{1,4}$ is an unknown vector function, $H(x) = col(0,0,0,h(x))$ is a given vector function, $A(x,\varepsilon) = A_0(x) + \varepsilon A_1(x)$ is known matrix. A method for solving this problem illustrated authors in more early works with stable turning point [2, 3].

Asymptotic forms of solutions for the system (1) are constructed in the form of the series

$$\tilde{Y_k}(x,t,\varepsilon) = \sum_{i=1}^{2} D_i(x,t,\varepsilon) + f(x,\varepsilon)\psi(t) + \varepsilon^{\gamma} g(x,\varepsilon)\psi'(t) + \omega(x,\varepsilon), \tag{2}$$

where $D_i(x,t,\varepsilon)$ is a matrix 4X4 multiply by the Airy functions $U_1(t)$, $U_2(t)$, and $\alpha_{ik}(x,\varepsilon)$, $\beta_{ik}(x,\varepsilon)$, $f_k(x,\varepsilon)$, $g_k(x,\varepsilon)$, $\omega_k(x,\varepsilon)$, $k = \overline{1,3}$ are analytic functions with reference to a small parameter and are infinitely differentiable functions of variable $x \in [-l;0]$ which determined.

Previous studies of the scalar linear ordinary differential equation for the third formal solution of the homogeneous vector equation (1) [1]

$$\varepsilon y'''(x,\varepsilon) + x\tilde{a}(x)y'(x,\varepsilon) + b(x)y(x,\varepsilon) = h(x), \tag{3}$$

have shown that structure of the solutions depends on the sign of the coefficients of $y(x)$ function and its first derivative $y'(x)$ which are the parts of the reduced equation:

$$L_0\omega(x) \equiv x\tilde{a}(x)\omega'(x) - b(x)\omega(x) = h(x). \tag{4}$$

In case with nonstable turning point we can't used a reduced system in the construction of one of the solutions of singularly perturbed system of differential equations with nonstable turning points. Instead we constructed asymptotic forms of partial solutions of equations (3).

The general solutions were writing a form:

$$f_0(x) = Cexp\Big\{\int_0^x \frac{b_1(x)}{2\tilde{a}(x)}dx\Big\}, \qquad g_0(x) = Cexp\Big\{\int_0^x \frac{b_2(x)}{2\tilde{a}(x)}dx\Big\}. \tag{5}$$

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Local boundedness of solutions to double phase parabolic equations

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We consider a class of parabolic equations with nonstandard growth condition and singular lower order term. Let Ω be a domain in $\mathbb{R}^n, T > 0$, set $\Omega_T = \Omega \times (0, T)$. We study solution to the equation

$$u_t - div\mathbb{A}(x, t, u, \nabla u) = f(x, t), (x, t) \in \Omega_T. \tag{1}$$

Throughout the paper we suppose that the functions $\mathbb{A}(\cdot, \cdot, u, \xi)$ are Lebesgue measurable for all $u \in \mathbb{R}^1, \xi \in \mathbb{R}^n$, $\mathbb{A}(x, t, \cdot, \cdot)$ are continuous for almost all $(x, t) \in \Omega_T$. We also assume that the following structure conditions are satisfied

$$\mathbb{A}(x, t, u, \xi)\xi \geq c_1(|\xi|^p + a(x, t)|\xi|^q), \ |\mathbb{A}(x, t, u, \xi)| \leq c_2(|\xi|^{p-1} + a(x, t)|\xi|^{q-1}), \tag{2}$$

where c_1, c_2 are positive constants, $a(x, t) \geq 0, a(x, t) \in C^{\alpha, \frac{\alpha}{2}}(\Omega_T)$ with some positive $\alpha \in (0, 1]$, $f \in L^1(\Omega_T)$, and

$$\frac{2n}{n+1} < p \leq q < p + \alpha. \tag{3}$$

The main goal is to establish local boundedness of solutions to equation (1) in terms of parabolic potential of the right-hand side. This fact is basically characterized by the different types of degenerate behavior according to the size of a coefficient $a(x, t)$ that determines the "phase". Indeed, on the set $a(x, t) = 0$ equation (1) has growth of order p with respect to the gradient (this is the "p-phase"), and at the same time this growth is of order q when $a(x, t) > 0$ (this is the "(p, q)-phase").

To describe our results let us remind the reader the definition of a weak solution to equation (1). For $\xi \in \mathbb{R}^n$ set $g_a(|\xi|) := |\xi|^{p-1} + a(x, t)|\xi|^{q-1}$ and $G_a(|\xi|) = |\xi| g_a(|\xi|)$. We will write $W^{1, G_a}(\Omega_T)$ for a class of functions which are weakly differentiable with

$\iint\limits_{\Omega_T} G_a(|\nabla u|)dxdt < \infty$. We say that u is a weak solution to (1) if $u \in V(\Omega_T) := C(0,T;L^2(\Omega)) \cap W^{1,G_a}(\Omega_T)$ and for any interval $(t_1,t_2) \subset (0,T)$ the integral identity

$$\int\limits_{\Omega} u\varphi dx \,|_{t_1}^{t_2} + \int\limits_{t_1}^{t_2}\int\limits_{\Omega}(-u\varphi_t + \mathbb{A}(x,t,u,\nabla u)\nabla\varphi)dxdt = \int\limits_{t_1}^{t_2}\int\limits_{\Omega}\varphi f dxdt$$

holds true for any testing function $\varphi \in \overset{0}{W}{}^{1,G_a}(\Omega_T)$ with $\varphi, \varphi_t \in L^\infty(\Omega_T)$.

Let $(x_0,t_0) \in \Omega_T$ for $\rho, \theta > 0$ and let $Q_{\rho,\theta}(x_0,t_0) := Q^-_{\rho,\theta}(x_0,t_0) \cup Q^+_{\rho,\theta}(x_0,t_0)$, $Q^-_{\rho,\theta}(x_0,t_0) := B_\rho(x_0) \times (t_0-\theta, t_0)$, $Q^+_{\rho,\theta}(x_0,t_0) := B_\rho(x_0) \times (t_0+\theta, t_0)$. For $m > \frac{2n}{n-1}$, $\rho > 0$ define

$$D_m(\rho; x_0,t_0) := \inf_{\tau>0}\left\{\frac{1}{\tau^{m-2}} + \rho^{-n}\iint\limits_{Q_{\rho,\rho^m\tau^{m-2}}(x_0,t_0)}|f|dxdt\right\}.$$

Now for $j = 0,1,2,...$ set $\rho_j := 2^{-j}\rho$. Following [1] we define the parabolic potential

$$P^f_m(\rho; x_0,t_0) := \sum_{j=0}^{\infty} D_m(\rho_j; x_0,t_0).$$

Our main result is the local boundedness of the solutions. As it has already mentioned before the behavior of the solution in a neighborhood of a point (x_0,t_0) depends on the value of the function $a(x_0,t_0)$. In what follows we will distinguish two cases: $\sup\limits_{Q_{\rho,\rho^2}(x_0,t_0)} a(x,t) \geq 2[a]_\alpha\rho^\alpha$ (so called (p,q)-phase) and $\sup\limits_{Q_{\rho,\rho^2}(x_0,t_0)} a(x,t) \leq 2[a]_\alpha\rho^\alpha$(so called p-phase), here $[a]_\alpha := \sup\limits_{\substack{(x,t),(y,\tau)\in\Omega_T\\(x,t)\neq(y,\tau)}} \frac{|a(x,t)-a(y,\tau)|}{(|x-y|+|t-\tau|)^\alpha}$.

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Asymptotic representations of some class of solutions with slowly varying derivatives of the second order differential equations with nonlinearities of different types

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The differential equation

$$y'' = \alpha_0 p(t)\varphi_0(y)\varphi_1(y'). \tag{1}$$

is considered. In this equation $\alpha_0 \in \{-1;1\}$, functions $p : [a,\omega[\to]0,+\infty[$, $(-\infty < a < \omega \le +\infty)$, and $\varphi_i : \Delta_{Y_i} \to]0,+\infty[$ $(i \in \{0,1\})$ are continuous, $Y_i \in \{0,\pm\infty\}$, Δ_{Y_i} is one-sided neighborhood of Y_i $(i \in \{0,1\})$.

We also suppose that the function φ_1 is a regularly varying function of index σ_1 as the argument tends to Y_1 ([3]), the function φ_0 is twice continuously differentiable on Δ_{Y_0} and satisfies the conditions

$$\varphi_0'(y) \neq 0 \ \text{ as } \ y \in \Delta_{Y_0}, \quad \lim_{\substack{y\to Y_0\\ y\in\Delta_{Y_0}}} \varphi_0(y) \in \{0,+\infty\}, \quad \lim_{\substack{y\to Y_0\\ y\in\Delta_{Y_0}}} \frac{\varphi_0(y)\varphi_0''(y)}{\left(\varphi_0'(y)\right)^2} = 1. \tag{2}$$

From the conditions (2) it follows (see [1]) that the function φ_0 and its derivative of the first order are rapidly varying functions as the argument tends to Y_0.

We consider the following class of solutions for the equation (1)

The solution y of the equation (1), that is defined on the interval $[t_0,\omega[\subset [a,\omega[$, is called $P_\omega(Y_0,Y_1,\lambda_0)$–solution $(-\infty \le \lambda_0 \le +\infty)$, if the next conditions take place

$$\lim_{t\uparrow\omega} y^{(i)}(t) = Y_i \ \ (i=0,1), \quad \lim_{t\uparrow\omega} \frac{(y'(t))^2}{y''(t)y(t)} = \lambda_0.$$

We say that a slowly varying as $z \to Y \quad (z \in \Delta_Y)$ function $\theta : \Delta_Y \to]0;+\infty[$ satisfies the condition S as $z \to Y$, if for any continuous differentiable normalized slowly varying as $z \to Y \quad (z \in \Delta_Y)$ function $L : \Delta_{Y_i} \to]0;+\infty[$ the next relation is valid

$$\theta(zL(z)) = \theta(z)(1+o(1)) \qquad \text{as } z \to Y, \quad (z \in \Delta_Y).$$

The following theorem is obtained

Theorem 1. *Let $\sigma_1 \neq 1$, the function $\varphi_1(y')|y'|^{-\sigma_1}$ satisfies the condition S as $y' \to Y_1 \quad (y' \in \Delta_{Y_1})$. Then, any $P_\omega(Y_0,Y_1,\pm\infty)$ – solution of the equation (1) can be represented as*

$$y(t) = \pi_\omega(t)L(t),$$

where $L : [t_0,\omega[\to R$ – is a such twice continuously differentiable function that

$$y_0^0\pi_\omega(t)L(t)>0,\quad L'(t)\neq 0\quad as\quad t\in[t_1,\omega[\ \ (t_0\leq t_1<\omega),$$

$$\lim_{t\uparrow\omega}L(t)\in\{0;\pm\infty\},\quad \lim_{t\uparrow\omega}\pi_\omega(t)L(t)=Y_0,\quad \lim_{t\uparrow\omega}\frac{\pi_\omega(t)L'(t)}{L(t)}=0.$$

In the case of the existence of a finite or infinite boundary

$$\lim_{t\uparrow\omega}\frac{\pi_\omega(t)L''(t)}{L'(t)},$$

the following relations take place

$$\lim_{t\uparrow\omega}\frac{\pi_\omega(t)L''(t)}{L'(t)}=-1,\quad \alpha_0L'(t)>0\quad as\quad t\in[t_1,\omega[(t_0\leq t_1<\omega),$$

$$p(t)=\frac{\alpha_0L'(t)}{\varphi_1(L(t))\varphi_0(\pi_\omega(t)L(t))}[1+o(1)]\quad as\quad t\uparrow\omega.$$

We introduce the notations

$$\mu_0=\text{sign}\varphi_0'(y),\quad \theta_1(y')=\varphi_1(y')|y'|^{-\sigma_1},$$

$$H(t)=\frac{L^2(t)\varphi_0'(\pi_\omega(t)L(t))}{L'(t)\varphi_0(\pi_\omega(t)L(t))},\quad q_1(t)=\left.\frac{\left(\frac{\varphi_0'(y)}{\varphi_0(y)}\right)'}{\left(\frac{\varphi_0'(y)}{\varphi_0(y)}\right)^2}\right|_{y=\pi_\omega(t)L(t)},$$

$$e_1(t)=1+\frac{\pi_\omega(t)L'(t)}{L(t)},\quad e_2(t)=2+\frac{\pi_\omega(t)L''(t)}{L'(t)}.$$

By (2) and (7), the following statements take place

1) $\lim\limits_{t\uparrow\omega}e_1(t)=\lim\limits_{t\uparrow\omega}e_2(t)=1\quad \lim\limits_{t\uparrow\omega}H(t)=\pm\infty,\quad \lim\limits_{t\uparrow\omega}q_1(t)=0,$

2) if the following limit exists $\lim\limits_{t\uparrow\omega}\frac{L(t)}{L'(t)}\cdot\frac{H'(t)}{|H(t)|^{\frac{3}{2}}}$, then $\lim\limits_{t\uparrow\omega}\frac{L(t)}{L'(t)}\cdot\frac{H'(t)}{|H(t)|^{\frac{3}{2}}}=0.$

Let's say that the condition N is executed, if for some continuously differentiable function $L(t):[t_0,\omega[\longrightarrow R(t_0\in[a,\omega[)$, which satisfies the conditions (5)-(7) and (9),the following representation is true

$$p(t)=\frac{\alpha_0L'(t)}{\varphi_1(L(t))\varphi_0(\pi_\omega(t)L(t))}[1+r(t)],$$

where $r(t):[t_0,\omega[\longrightarrow]-1;+\infty[$ – continuous function that tends to zero as $t\uparrow\omega$.

The following theorem is true

Theorem 2. *Let $\sigma_1\neq 1$, the function θ_1 satisfies the condition S, the condition N is fulfilled and*

$$\lim_{t\uparrow\omega}\frac{\pi_\omega(t)L'(t)}{L(t)}|H(t)|^{\frac{1}{2}}=\pm\infty.\tag{17}$$

Then if $\alpha_0\mu_0 > 0$, *then then the differential equation (1) has a one-parameter family of* $P_\omega(Y_0, Y_1, \pm\infty)$*-solutions, for each of which the following asymptotic representation take place as* $t \uparrow \omega$.

$$y(t) = \pi_\omega(t) \cdot L(t) + \frac{\varphi_0'(\pi_\omega(t)L(t))}{\varphi_0(\pi_\omega(t)L(t))} \cdot o(1),$$

$$y'(t) = [L(t) + \pi_\omega(t) \cdot L'(t)] \cdot [1 + |H(t)|^{-\frac{1}{2}} \cdot o(1)].$$

In the work we have found the necessary and sufficient conditions for the existence of $P_\omega(Y_0, Y_1, \pm\infty)$–solutions of the equation (1) and asymptotic representations of such solutions and their first order derivatives as $t \uparrow \omega$. The results are obtained by modifying the methods of studying the asymptotic properties of $P_\omega(Y_0, Y_1, \pm\infty)$–solutions that have been developed for some special type of equation (1) with $\varphi_1 \equiv 1$. (see [2]).

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Natural invariants and Stochastic First Integrals for the generalized Ito's equations

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Various conservation laws, such as energy, weight, impulse, impulse moment, etc., are the basis of invariants and first integrals. For example, if an enumerable collection of initial solutions for the same dynamical equation is connected to points which are similar to particles, then the number of this points is a conservative value since the conditions of the existence and uniqueness of the solution are fulfilled. The limit state of this representation is a density $\rho_l(x;t)$ of the number of these points and conservation of the integral for it in real space:

$$\int_{\mathrm{R}^n} \rho_l(x;t)\,dx = \int_{\mathrm{R}^n} \rho_l(x)\,dx = 1,\ \forall\, t \geq 0, \quad \rho_l(x) \geq 0\ \forall x \in \mathbb{R}^n.$$

A function $\rho_l(x;t)$ with this property is a kernel's function of an invariant integral (natural invariant). The number of such independent kernels $\rho_l(x;t)$ is not more than $n+1$. By imposing certain restrictions, a partial differential equation for the kernel function can be obtained. Let $x(t;x(0))$ be a solution of the classical stochastic differential equations (Wiener and Poisson perturbations). The equations for the kernels can be constructed using the rules of Ito's stochastic differentiation, the generalized Ito-Ventzell formula and the following requirement:

$$d_t\rho_l(x(t;x(0));t)J(t) = 0, \quad J(t) = J(t;x(0)),\ \forall\, t \geq 0,\ \forall\, x(0) \in \mathbb{R}^n,$$

where $J(t)$ is the Jacobian of the transformation that connected with the dynamic process $x(t;x(0)) \in \mathrm{R}^n$.

The concept of a first integral for solution of the deterministic dynamical system is the fundamental concept of analytical mechanics. For stochastic differential equations (SDE), similar concepts also exist. They are a first integral for Ito's SDE (Doobko,1978), a first forward integral and a first backward integral for Ito's SDE (Krylov and Rozovsky, 1982), a stochastic first integral for generalized Ito's SDE (GSDE) (Doobko, 2002).

A direct transfer of the concept of a first integral from deterministic systems to stochastic systems is impossible. However, we can check up that

$$d_t\rho_l(x(t;x(0));t)/\rho_k(x(t;x(0));t) = 0,\ \forall\, t \geq 0,\ \forall\, x(0) \in \mathbb{R}^n.$$

Hence $u_{l,k}(x;t) = \rho_l(x;t)/\rho_k(x;t)$ is a stochastic first integral. By means of this equality the partial differential equation for $u_{l,k}(x;t)$ is constructed.

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Extension and Lagrange stability of solutions of degenerate differential-operator equations

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Consider a differential-operator equation (DOE) of the form

$$\frac{d}{dt}[A(t)x(t)] + B(t)x(t) = f(t, x(t)) \tag{1}$$

with the initial condition

$$x(t_0) = x_0, \tag{2}$$

where A, $B \in C([t_+, \infty), L(\mathbb{R}^n))$, $f \in C([t_+, \infty) \times \mathbb{R}^n, \mathbb{R}^n)$, and $t_0 \geq t_+ \geq 0$. Generally, the operator $A(t)$ is degenerate ($B(t)$ may also be degenerate) and in this case the DOE is called degenerate. Degenerate DOEs are also called differential-algebraic equations (DAEs). A function $x \in C([t_0, t_1), \mathbb{R}^n)$ is called a solution of the initial value problem (IVP) (1), (2) on some interval $[t_0, t_1) \subseteq [t_+, \infty)$, if $Ax \in C^1([t_0, t_1), \mathbb{R}^n)$, and $x(t)$ satisfies the equation (1) on $[t_0, t_1)$ and the initial condition (2). A solution $x(t)$ of the IVP (1), (2) is called global if it exists on the whole interval $[t_0, \infty)$, and Lagrange stable if it is global and bounded, i.e., $\sup\limits_{t \in [t_0, \infty)} \|x(t)\| < \infty$. A solution $x(t)$ of the IVP (1), (2) is called Lagrange unstable if it exists on some finite interval $[t_0, T)$ and is unbounded (the solution has a finite escape time), i.e., there exists $T < \infty$ such that $\lim\limits_{t \to T-0} \|x(t)\| = \infty$. The equation (1) is Lagrange stable if every solution of the IVP (1), (2) is Lagrange stable.

It is assumed that the operator pencil $\lambda A(t) + B(t)$ is regular for every $t \geq t_+$, and there exist functions $C_1 \colon [t_+, \infty) \to (0, \infty)$, $C_2 \colon [t_+, \infty) \to (0, \infty)$ such that for all $t \in [t_+, \infty)$ and $|\lambda| \geq C_2(t)$ the condition $\|(\lambda A(t) + B(t))^{-1}\| \leq C_1(t)$ holds. For the degenerate DOE (1) with the stationary operators A, B, the theorem on the unique global solvability and the theorems on the Lagrange stability were obtained in [1] and [2] respectively. In this work, we obtain the theorems on the unique global solvability and Lagrange stability of the DOE (1) with the non-stationary operators $A(t)$, $B(t)$. To prove them, we apply the spectral projectors of Riesz type, the implicit function theorems, the method of the extension of solutions and differential inequalities with La Salle functions. To prove the global solvability we do not use constraints of a global Lipschitz condition type. This allows solving more general classes of applied problems. The application of the obtained results to the investigation of mathematical models of nonlinear radio engineering filters are discussed.

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Mechanics of blood flow in the aorta

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The blood movement in vessels in terms of hydrodynamics is considered ([1], [2]).

We study the law of changes in blood pressure $p(t)$ in the human aorta during the complete cardiac cycle ([3]). It is shown that under some assumptions the pressure change is described by the equation

$$\frac{\partial p}{\partial t} = \frac{1}{k}\left[Q(t) - \frac{p(t)}{\omega}\right], \qquad (1)$$

where $k > 0$ – aortic wall elasticity, $\omega > 0$ – hydraulic resistance of the microvascular system, $Q(t)$ – volumetric rate of blood flow from the heart to the aorta. We obtained explicit solutions of (1) in the following cases:

a) with parabolic change of $Q(t)$ in the systolic phase;

b) with $Q(t) \equiv 0$ in the diastolic phase.

We established that growth of the hydraulic resistance ω or growth of the elasticity k leads to the blood's pressure increasing in the aorta after closing of the aortic valve. In particular, the pressure is increasing at the end of the diastolic phase.

It was found that growth of the hydraulic resistance ω or growth of the elasticity k means the velocity decreasing of blood pressure in the aorta in diastolic phase.

The pulse wave propagation process was also studied. We have a condition of the pulse wave propagation without reflection. A normal functioning of the human circulatory system is difficult without this condition. In particular, an aneurysm can develop ([4]).

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Identification of the unknown coefficients in the degenerate parabolic equation

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In the rectangle $Q_T = \{(x,t) : 0 < x < h, 0 < t < T\}$ we consider the inverse problem of simultaneous identification of two time-dependent coefficients $a = a(t), a(t) > 0,\ t \in [0,T]$ and $b = b(t)$ in the parabolic equation

$$\psi(t)u_t = a(t)u_{xx} + b(t)u_x + c(x,t)u + f(x,t) \tag{1}$$

with initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{2}$$

boundary conditions

$$u_x(0,t) = \mu_1(t), \quad u_x(h,t) = \mu_2(t), \quad t \in [0,T] \tag{3}$$

and overdetermination conditions

$$u(0,t) = \mu_3(t), \quad t \in [0,T], \tag{4}$$

$$\int_0^h u(x,t)dx = \mu_4(t), \quad t \in [0,T]. \tag{5}$$

It is known that h, T are the positive numbers and $\psi = \psi(t)$ is a monotone increasing function such that $\psi(t) > 0,\ t \in (0,T]$ and $\psi(0) = 0$. We investigate the case of weak degeneration when $\lim\limits_{t\to+0} \int_0^t \frac{d\tau}{\psi(\tau)} = 0$.

Under the solution to the problem (1)-(5) we understand the triplet of functions (a,b,u) such that $(a,b,u) \in (C[0,T])^2 \times C^{2,1}(\overline{Q}_T),\ a(t) > 0, t \in [0,T]$ and verifies the equation (1) and conditions (2)-(5).

Using the Green functions of the first and second boundary value problems for the heat equation

$$\psi(t)u_t = a(t)u_{xx}$$

we reduce the problem (1)-(5) to the equivalent system of equations. Applying the Schauder fixed-point theorem to this system it is established conditions of the existence of the solution to the problem (1)-(5) on a small segment of time.

The proof of the uniqueness of the solution to the problem (1)-(5) is based on the properties of the solutions to the homogeneous integral Volterra equations of the second kind with integrable kernels.

Note that the conditions of the solvability to the inverse problems of determination of the coefficient $a = a(t), a(t) > 0, t \in [0, T]$ and $b = b(t)$ in equation (1) separately are established in [1], [2] respectively. The problem of simultaneous identification of the coefficients $a = a(t), a(t) > 0, t \in [0, T]$ and $b = b(t)$ in the parabolic equation with weak power degeneration under Dirichlet boundary conditions and known values of the heat flux and heat moment is founded in [3].

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The Riemann-Hilbert approach to the Cauchy problem for the modified Camassa-Holm equation

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We study the Cauchy problem for the modified Camassa–Holm (mCH) equation

$$\begin{array}{lll} m_t + ((u^2 - u_x^2)m)_x = 0, \quad m = u - u_{xx}, & & -\infty < x < \infty,\ t \geq 0, \\ u(x,0) = u_0(x), & & -\infty < x < \infty, \end{array} \tag{1}$$

in the class of functions satisfying, for all $t \geq 0$, the nonzero boundary conditions: $u(x,t) \to c$ as $|x| \to \infty$, where $c \in \mathbb{R}$ is a constant.

The mCH equation was introduced as a new integrable system by Fuchssteiner [1] and Olver and Rosenau [2]. It also arises in the theory of nonlinear water waves as a model equation and from an (intrinsic) arc-length preserving invariant planar curve flow in Euclidean geometry.

We present the inverse scattering transform (IST) approach for the mCH equation using the formalism of 2×2 matrix Riemann–Hilbert problems formulated in the complex plane of the spectral parameter (cf. [3]). This approach is applied to the Lax pair of the mCH equation:

$$\begin{cases} \Phi_x(x,t,\lambda) = U(x,t,\lambda)\Phi(x,t,\lambda) \\ \Phi_t(x,t,\lambda) = V(x,t,\lambda)\Phi(x,t,\lambda) \end{cases},$$

where

$$\begin{aligned} U &= \begin{pmatrix} \frac{-1}{2} & \frac{\lambda m}{2} \\ \frac{-\lambda m}{2} & \frac{1}{2} \end{pmatrix}, \\ V &= \begin{pmatrix} \lambda^{-2} + \frac{(u^2-u_x^2+2u)}{2} & -\lambda^{-1}(u-u_x+1) - \frac{\lambda(u^2-u_x^2+2u)m}{2} \\ \lambda^{-1}(u+u_x+1) + \frac{\lambda(u^2-u_x^2+2u)m}{2} & -\lambda^{-2} - \frac{(u^2-u_x^2+2u)}{2} \end{pmatrix}. \end{aligned}$$

We construct a parametric representation of the smooth solution of problem (1) in terms of the solution of an associated Riemann–Hilbert problem, which can be efficiently used for further studying the properties of the solution. Particularly, using the proposed formalism, we describe regular as well as non-regular (cuspon and loop-shaped) one-soliton solutions [4] corresponding to the Riemann–Hilbert problems with trivial jump conditions and appropriately chosen residue conditions.

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The existence of H-solution to optimal control problem for degenerate elliptic variation inequality

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We investigate the optimal control problem for degenerate elliptic variation inequality. It is known, that dealing with studying of optimization problems with such object of control leads us to such problem as Lavrentieff phenomenon, non-uniqueness of definition of the solution to variation inequality and, as the consequence, non-uniqueness of definition of the optimal solution. In [1] the initial degenerate problem is reduced to equivalent (in some sense) problem in “classical” Sobolev space and it is proved its solvability in the case when the weight function is the function of potential type. Similarly such problems are solved in [2, 3] for degenerate parabolic variation inequality. As part of the given investigation we propose the alternative approach to studying of the solvability problem for the mentioned optimal control problem for degenerate elliptic variation inequality. Namely, similarly to [4], where the optimization problem for degenerate non-linear monotone variation inequality with control in coefficients is investigated, we introduce the class of so-called H-admissible solutions. Thus, in [5], using the direct method of calculus of variations, we justified the H-solvability for the optimization problem for degenerate elliptic variation inequality with control in right-hand part.

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Classical solution to the Poisson's equation

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Consider the Poisson's equation [1, c. 276]

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = -f(x,y), \tag{1}$$

and such function:

$$u_1(x,y) = \frac{i}{2}\int_0^y d\eta \int_{x-i(y-\eta)}^{x+i(y-\eta)} f(\xi,\eta)d\xi, \quad i = \sqrt{-1}. \tag{2}$$

Find the partial derivatives of the first and second orders of the function $u_1(x,y)$. On the basis of formula (2) we find

$$\frac{\partial u_1}{\partial x} = \frac{i}{2}\int_0^y \left(f(x+i(y-\eta),\eta) - f(x-i(y-\eta),\eta)\right) d\eta. \tag{3}$$

Introduce the notation: $\alpha(x,y,i,\eta) = x + i(y-\eta)$; $\quad \beta(x,y,i,\eta) = x - i(y-\eta)$. For $f(x,y) \in C^{1,0}$ we have

$$\frac{\partial^2 u_1}{\partial x^2} = \frac{i}{2}\int_0^y \left(\frac{\partial f(\alpha(x,y,i,\eta),\eta)}{\partial \alpha} - \frac{\partial f(\beta(x,y,i,\eta),\eta)}{\partial \beta}\right) d\eta;$$

$$\frac{\partial u_1}{\partial y} = \frac{i}{2}\int_0^y \left(if(x+i(y-\eta),\eta) + if(x-i(y-\eta),\eta)\right) d\eta;$$

$$\frac{\partial^2 u_1}{\partial y^2} = \frac{i}{2}\int_0^y \left(i^2\ \partial f(\alpha(x,y,i,\eta),\eta) - i^2\ \partial f(\beta(x,y,i,\eta),\eta)\right) d\eta - f(x,y).$$

So,

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = -f(x,y).$$

This signifies that the function $u_1(x,y)$ defined by the formula (3) is a partial solution to the equation (1).

Similarly we prove, the function

$$u_2(x,y) = \frac{i}{2}\int_y^\pi d\eta \int_{x+i(y-\eta)}^{x-i(y-\eta)} f(\xi,\eta)d\xi, \quad i = \sqrt{-1}, \tag{4}$$

for $f(x,y) \in C^{1,0}$, is a partial solution to the equation (1).

Theorem. If the function $f(x,y) \in C^{1,0}$, then the function

$$u(x,y) = \frac{i}{4}\int_0^y d\eta \int_{x-i(y-\eta)}^{x+i(y-\eta)} f(\xi,\eta)d\xi + \frac{i}{4}\int_y^\pi d\eta \int_{x+i(y-\eta)}^{x-i(y-\eta)} f(\xi,\eta)d\xi. \tag{5}$$

is the classical ($u \in C^{2,2}$) solution to the equation (1).

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Operator research of classical solutions to boundary-value problems for hyperbolic second order equations

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Statement of the problem: to find a classical solution to the hyperbolic second order equation

$$u_{tt} - u_{xx} = g(x,t), \quad 0 \le x \le \pi, \quad t \in \mathbb{R}, \tag{1}$$

which satisfies the boundary conditions

$$u(0,t) = u(\pi,t) = 0, \quad t \in \mathbb{R}. \tag{2}$$

We have proved that the classical solution to the problem (1), (2) is the function [1, 2]

$$u(x,t) = (Rg)(x,t) \equiv (Sg)(x,t) + \left(\widetilde{S}g\right)(x,t), \tag{3}$$

where

$$(Sg)(x,t) = -\frac{1}{4}\int_0^x d\xi \int_{t-x+\xi}^{t+x-\xi} g(\xi,\tau)\,d\tau - \frac{1}{4}\int_x^\pi d\xi \int_{t+x-\xi}^{t-x+\xi} g(\xi,\tau)\,d\tau; \tag{4}$$

$$\left(\widetilde{S}g\right)(x,t) = \frac{\pi - x}{4\pi}\int_0^\pi d\xi \int_{t-\xi}^{t+\xi} g(\xi,\tau)\,d\tau - \frac{x}{4\pi}\int_0^\pi d\xi \int_{t-\pi+\xi}^{t+\pi-\xi} g(\xi,\tau)\,d\tau; \tag{5}$$

$g(x,t) \in C^{0,1}$, $\quad C^{0,1}-$ is the space of functions of two variables, continuous and bounded together with the derivative of t, defined on the set $[0,\pi] \times \mathbb{R}$.

For the boundary-value problem

$$u_{tt} - u_{xx} = f(x,t), \quad x \in \mathbb{R}, \quad 0 \le t \le \pi, \tag{6}$$

$$u(x,0) = u(x,\pi) = 0, \quad x \in \mathbb{R}, \tag{7}$$

we have proved (it's a new result) that the function

$$u(x,t) = (Bf)(x,t) \equiv (Lf)(x,t) + \left(\widetilde{L}f\right)(x,t), \tag{8}$$

where

$$(Lf)(x,t) = \frac{1}{4}\int\limits_0^t d\tau \int\limits_{x-t+\tau}^{x+t-\tau} f(\xi,\tau)\,d\xi + \frac{1}{4}\int\limits_t^\pi d\tau \int\limits_{x+t-\tau}^{x-t+\tau} f(\xi,\tau)\,d\xi; \tag{9}$$

$$\left(\widetilde{L}f\right)(x,t) = -\frac{\pi - t}{4\pi}\int\limits_0^\pi d\tau \int\limits_{x-\tau}^{x+\tau} f(\xi,\tau)\,d\xi - \frac{t}{4\pi}\int\limits_0^\pi d\tau \int\limits_{x-\pi+\tau}^{x+\pi-\tau} f(\xi,\tau)\,d\xi; \tag{10}$$

$f(x,t) \in C^{1,0}$, $\quad C^{1,0}-$ is the space of functions of two variables, continuous and bounded together with the derivative of x, defined on the set $\mathbb{R} \times [0,\pi]$. It is easy to verify that both classical solutions (3)-(5) and (8)-(10) satisfy the boundary conditions (2) and (7). The class of functions for which solutions (3) and (8) are classical solutions to the equations (1) and (6) is established. We've considered examples for which the formal solution to such boundary-value problem

$$u_{tt} - u_{xx} = cu, \tag{11}$$

$$u(0,t) = u(\pi,t) = 0, \quad t \in \mathbb{R}. \tag{12}$$

can be easily found by using the method of separating variables (by the Fourier method) and is represented by Fourier series

$$u(x,t) = \sum_{k=1}^{\infty} \big(A_k \cos\omega_k t + B_k \sin\omega_k t\big) \sin kx, \quad \omega_k = \sqrt{k^2 - c}, \tag{13}$$

And also for the boundary-value problem

$$u_{tt} - u_{xx} = cu, \tag{14}$$

$$u(x,0) = u(x,\pi) = 0, \quad x \in \mathbb{R}, \tag{15}$$

its formal solution is represented by Fourier series such as

$$u(x,t) = \sum_{k=1}^{\infty} \big(A_k \cos\omega_k x + B_k \sin\omega_k x\big) \sin kt, \quad \omega_k = \sqrt{k^2 + c}. \tag{16}$$

The solution (13) (as well as the solution (16)) is called the formal solution because it can not be differentiated by t and x twice. There are no conditions for this. And, therefore, there are two questions:

1) What to do when there are no initial conditions by setting boundary tasks (1)-(2) or (11)-(12) or (14)-(15)?

2) When is a classical solution to equations (1) or (6)?

The answer to both questions is given in our abstract. Also, the obtained solutions are used to study the T-periodic solutions to hyperbolic equations [4–6].

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Application of the integral transformations for the uniqueness theorems

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The integral equation of the special form is considered in this paper.

Let $0 < r < R$, $H \in C^s[r-R, R-r]$, $H(-x)(-1)^s = H(x)$ and

$$\int_{-\pi}^{\pi} H(x \cos t) \cos st dt \equiv 0.$$

Then H is a polynomial of degree $s-1$ with $s \geq 1$ and $H \equiv 0$ with $s = 0$ on the segment $[r-R, R-r]$.

The analogues of the classical uniqueness theorems with the mean value conditions can be constructed using the similar results (see [3]). We consider the radial case and represent the main function by the integral of $f \epsilon C^s(-R, R)$.

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On the stability of the rotation of the Lagrange top with an ideal liquid under the action of the dissipative and two constant moments

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In the talk, we consider the rotation around a fixed point of a heavy dynamically symmetric solid with an arbitrary axisymmetric cavity completely filled with an ideal incompressible liquid. In addition to gravity, a solid body is affected by a dissipative moment

$$\overrightarrow{M}_d = -D\overrightarrow{\omega} \quad (D = \mathrm{diag}(D_1, D_2, D_3), \; D_i > 0, \; i = \overline{1,2})$$

simulating a resisting medium, a constant moment $\overrightarrow{M}_q = Q\overrightarrow{k}$ in a non-inertial coordinate system and a constant moment $\overrightarrow{M}_p = P\overrightarrow{\gamma}$ in the inertial coordinate system. Here $\overrightarrow{k}$ and $\overrightarrow{\gamma}$ are the unit vectors directed along the axis of symmetry of the top and in the ascending vertical respectively, Q and P are arbitrary constants.

The stability of a uniform rotation in a resisting medium of the Lagrange top with an ideal liquid is studied with regard to two constant moments under the assumption that in undisturbed motion a solid and a liquid rotate as one with angular velocity $\omega = (Q+P)/D_3$. In the absence of the relative liquid motion ("frozen liquid"), this problem was considered in [1].

Let $\omega = (Q+P)/D_3$ and the top is affected by the overturning moment ($\Gamma > 0$). In this case, the characteristic equation of perturbed motion is

$$A + \frac{iC\omega + D_i}{\lambda - i\omega} - \frac{\Gamma - iP}{(\lambda - i\omega)^2} - \lambda \sum_{n=1}^{æ} \frac{E_n}{\lambda - \omega\lambda_n} = 0, \tag{1}$$

where A and C are the equatorial and axial moments of inertia of a solid and a liquid respectively, λ_n are the natural frequencies of the uniformly rotating ideal liquid in the axisymmetric cavity, $E_n = 2a_n^2/N_n^2 > 0$. The definitions of the other quantities are given in [2].

Taking into account the fundamental tone of the liquid oscillations ($n = 1$), equation (1) takes the form

$$a_3\lambda^3 + (a_2 + ib_2)\lambda^2 + (a_1 + ib_1)\lambda + a_0 + ib_0 = 0, \tag{2}$$

where

$$\begin{gathered} a_3 = A - E_1 = A^* > 0, a_2 = D_1 > 0, b_2 = (C - A\lambda_1 - 2A^*)\omega, \\ a_1 = [C - A^* + (C - 2A)\lambda_1]\omega^2 - \Gamma, b_1 = P - (1+\lambda_1)D_1\omega, \\ a_0 = (P - D_1\omega)\lambda_1\omega, b_0 = [\Gamma - (C - A)\omega^2]\lambda_1\omega. \end{gathered}$$

For the existence of asymptotically stable solutions, it is necessary and sufficient that a fifth-order matrix composed of the coefficients of the polynomial (2) be inner positive [3], and the matrices Δ_3 and Δ_5 were positively identified :

$$-D_1^2\Gamma + E_1\lambda_1(\lambda_1 - 1)D_1^2\omega^2 + (C - E_1\lambda_1)PD_1\omega - A^*P^2 > 0, \tag{3}$$

$$\omega[(\lambda_1 - 1)\omega D_1 + P]D_1^2\Gamma - P[(\lambda_1 - 1)C\omega^2 D_1^2 + (A\lambda_1 - A^* - C)PD_1\omega - A^*P^2] < 0. \tag{4}$$

Asymptotic stability of the solution $\omega = (Q-P)/D_3$ provided that the top is affected by the restoring moment ($\Gamma < 0$) follows from inequalities (3)-(4), if we replace P by $-P$.

Based on the analytical and numerical studies of inequalities (3)-(4) the following conclusions were made:

1. In the absence of a constant moment in the inertial coordinate system ($P = 0$) uniform rotation will not be asymptotically stable under the action of the tilting moment

($\Gamma > 0$) and will be under the action of the restoring moment ($\Gamma < 0$) and $\lambda_1 > 1$. For the free top ($\Gamma = 0$), the stability condition is determined by the inequality $\lambda_1 > 1$.

2. The stability conditions for the not free Lagrange top ($\Gamma \neq 0$) are determined by the cubic and square inequalities with respect to x ($x = D_1/D_3 > 0$), and do not depend on the sign of the value P. For the free Lagrange top ($\Gamma = 0$), these conditions are already determined by two square inequalities, and the quantities do not depend on P ($P \neq 0$).

3. For an ellipsoidal cavity, the asymptotic stability of uniform rotation will be only for a preloaded ellipsoidal cavity. It is shown that the stability regions decrease with an increase in the equatorial moment of inertia of a solid, and they increase with an increase in the axial moment of inertia of a solid.

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One-dimensional diffusion processes with moving membrane and classical potentials

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The report deals with the problem of construction of Feller semigroup for one-dimensional inhomogeneous diffusion processes with membrane placed at a point whose position on the real line is determined by a given function that depends on the time variable. It is assumed that in the inner points of the half-lines separated by a membrane the desired process must coincide with the ordinary diffusion processes given there, and its behavior on the common boundary of these regions is determined by one of the variants of the general Feller-Wentzell conjugation condition (see [1, 2, 3]). In the case considered here this condition has the nonlocal character. It contains only the terms which correspond to the property of partial reflection of process and to the possibility of its exit from the boundary of the domain by jumps. This problem is often called the problem of pasting together two diffusion processes on a line or the problem of construction of mathematical model of physical phenomenon of diffusion in medium with membrane (see [4, 5]).

In order to study the described problem we use analytical methods. Such an approach allows us to determine the desired operator family using the solution of the corresponding problem of conjugation for a linear parabolic equation of the second order (the Kolmogorov backward equation) with discontinuous coefficients. This solution is constructed by the boundary integral equations method under the assumption that the coefficients of the equation satisfy the Hölder condition with a nonzero exponent, the initial function is bounded and continuous on the whole real line, and the parameters characterizing the Feller-Wentzell conjugation condition and the curve defining the common boundary of the domains, where the equation is given, satisfies the Hölder condition with exponent greater than $\frac{1}{2}$.

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Pointwise estimates of weak solutions for quasi-linear elliptic equations of divergence type with lower terms

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We consider the quasi-linear elliptic equations of divergence type with non-standard growth conditions and lower term $b(x, u, \nabla u)$

$$-\mathrm{div} A(x,\, \nabla u) + b(x, u, \nabla u) = f(x), \tag{1}$$

where $f(x) \in L^1(\Omega)$. Function $A(x, \xi) : \Omega \times R^n \to R^n$ satisfies the following conditions:
1) $A(x,\, \xi)$ satisfies the Carathéodory condition,
2) $A(x, \xi) \geq \mu_1 g(\xi)|\xi|$,
3)$|A(x, \xi)| \leq \mu_2 g(\xi)$,
4)$|b(x, u, \xi)| \leq c_1 g(u) + c_2 g(|\xi|)$,
with some constants $\mu_1,\, \mu_2, c_1,\, c_2 > 0$.

$$g \in C(\mathbb{R}^{\Bbbk}_{+}), \left(\frac{t}{\tau}\right)^{p-1} \leq \frac{g(t)}{g(\tau)} \leq \left(\frac{t}{\tau}\right)^{q-1}, 0 < \tau \leq t, 1 < p \leq 1 < n. \tag{2}$$

The model example of the equation (1) is

$$-\mathrm{div}\big(g(|\nabla u|)\frac{\nabla u}{|\nabla u|}\big) + b(x, u, \nabla u) = f(x), \tag{3}$$

We assume that $W^{1,G}(\Omega)$ is a functional space of weak solutions of the equation (1) defined by the following definition of the weak solution equation (1).

Definition. We say that u is a weak solution to Eq.(1), if $u \in W^{1,G}(\Omega)$, and it satisfies the integral identity

$$\int\limits_{\Omega} \big(g(|\nabla u|)\frac{\nabla u}{|\nabla u|}\big)\nabla\varphi\, dx + \int\limits_{\Omega} b(x, u, \nabla u)\varphi\, dx = \int\limits_{\Omega} f\varphi\, dx, \tag{4}$$

for all $\varphi \in \overset{0}{W}{}^{1,G}(\Omega)$.

Theorem 1. *Let $u \in W^{1,G}(\Omega) \cap L^{\infty}-$ be a nonnegative weak solution to Eq. (1) and conditions 1)-4), (2) are fulfilled. Then there exist constants $c_1,\, c_2 > 0$, depending only on $p,\, q,\, n, \mu_1, \mu_2$ such that, for almost all $x_0 \in \Omega, B_{4\rho}(x_0 \subset \Omega)$, the following estimate holds:*

$$u(x_0) \leq c_1 W^{f}_{1,g}(x_0,\, \rho) \tag{5}$$

or

$$g^{1+\lambda_0}\left(\frac{u(x_0)}{\rho}\right) \leq c_2 \rho^{-n} \int\limits_{B_{\rho_j}(x_0)} g^{1+\lambda_0}\left(\frac{u}{\rho}\right) dx \tag{6}$$

where $0 < \lambda_0 < \frac{1}{n-1}$.

Theorem 2. *Let* $u \in W^{1,G}(\Omega) \cap L^{\infty}-$ *be a nonnegative weak solution to (1),*$f \geq 0$ *and conditions 1)-4), (2) are fulfilled. Then there exist constants* c_3, $c_4 > 0$, *depending only on* p, q, n, μ_1, μ_2 *such that, for almost all* $x_0 \in \Omega, B_{4\rho}(x_0 \subset \Omega)$, *the following estimate holds:*

$$c_3 W^f_{1,g}(x_0, \rho) \leq u(x_0) \leq c_4 \inf_{B_{\rho(x_0)}} u + c_4 W^f_{1,g}(x_0, 2\rho).$$

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Unique solvability of the nonlocal problem with integral condition for second order differential equations

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Let $H(([T_1, T_2] \cup [T_3, T_4]) \times \mathbb{R}_+)$ be a class of entire function, K_L be a class of quasi-polynomials of the form

$$\varphi(x) = \sum_{i=1}^{n} Q_i(x) e^{\alpha_i x},$$

where $Q_i(x)$ are given polynomials, $L \subseteq \mathbb{C}$, $\alpha_k \neq \alpha_l$ for $k \neq l$.

Each quasi-polynomial defines a differential operator $\varphi\left(\frac{\partial}{\partial \lambda}\right)$ of finite order on the class of certain function in the form

$$\sum_{j=1}^{m} Q_{ji}\left(\frac{\partial}{\partial \lambda}\right) \exp\left[\alpha_i \frac{\partial}{\partial \lambda}\right] \Bigg|_{\lambda=0}.$$

In the strip $\Omega = \{(t, x) \in \mathbb{R}^2 : t \in (T_1, T_2) \cup (T_1, T_2), x \in \mathbb{R}\}$ we consider the problem with integral conditions

$$\frac{\partial^2 U}{\partial t^2} + a\left(\frac{\partial}{\partial x}\right)\frac{\partial U}{\partial t} + b\left(\frac{\partial}{\partial x}\right)U(t, x) = 0 \tag{1}$$

satisfies nonlocal-integral conditions

$$\int\limits_{T_1}^{T_2} U(t,x)dt + \int\limits_{T_3}^{T_4} U(t,x)dt = \varphi_1(x); \quad t \in [T_1,T_2] \cup [T_3,T_4]), \tag{2}$$

$$P\left(\frac{\partial}{\partial x}\right)\frac{\partial U}{\partial t}\Bigg|_{t=T_1} + Q\left(\frac{\partial}{\partial x}\right)\frac{\partial U}{\partial t}\Bigg|_{t=T_2} + \int\limits_{T_1}^{T_2} tU(t,x)dt + \int\limits_{T_3}^{T_4} tU(t,x)dt = \varphi_2(x), \tag{3}$$

where $a\left(\frac{\partial}{\partial x}\right)$, $b\left(\frac{\partial}{\partial x}\right)$ are differential expressions with entire functions $a(\lambda), b(\lambda) \neq const$, $P\left(\frac{\partial}{\partial x}\right), Q\left(\frac{\partial}{\partial x}\right)$ are given differential polynomials. $M_1(t,\lambda), M_2(t,\lambda)$ are solution of the equations

$$\left[\frac{d^2}{dt^2} + a(\lambda)\frac{d}{dt} + b(\lambda)\right] M_m(t,\lambda) = 0$$

satisfies nonlocal conditions

$$\int_{T_1}^{T_2} M_1(t,\lambda)dt + \int_{T_3}^{T_4} M_1(t,\lambda)dt = 1, \tag{4}$$

$$P(\lambda)\frac{d}{dt}M_1(t,\lambda)\Bigg|_{t=T_1} + Q(\lambda)\frac{d}{dt}M_1(t,\lambda)\Bigg|_{t=T_2} + \int_{T_1}^{T_2} tM_1(t,\lambda)dt + \int_{T_3}^{T_4} tM_1(t,\lambda)dt = 0,$$

$$\int_{T_1}^{T_2} M_2(t,\lambda)dt + \int_{T_3}^{T_4} M_2(t,\lambda)dt = 0, \tag{5}$$

$$P(\lambda)\frac{d}{dt}M_2(t,\lambda)\Bigg|_{t=T_1} + Q(\lambda)\frac{d}{dt}M_2(t,\lambda)\Bigg|_{t=T_2} + \int_{T_1}^{T_2} tM_2(t,\lambda)dt + \int_{T_3}^{T_4} tM_2(t,\lambda)dt = 1.$$

Denote by

$$P = \{\lambda \in \mathbb{C} : \Delta(\lambda) = 0\}, \tag{6}$$

where $\Delta(\lambda)$ is a determinant of system (4), (5).

Theorem. Let $\varphi_1(x), \varphi_2(x) \in K_{\mathbb{C}\setminus P}$, where P is set (6) . Then in the class of functions $K_{\mathbb{C}\setminus P}$ there exists a unique solution of the problem (1), (2), (3), which can be determined by the formula

$$U(t,x) = \varphi_1\left(\frac{\partial}{\partial\lambda}\right)\left\{M_1(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0} + \varphi_2\left(\frac{\partial}{\partial\lambda}\right)\left\{M_2(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}.$$

By means of the differential symbol method [1] we construct a solution of the problem (1), (2), (3). This problem continues the works [2], [3].

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On the solvability of third boundary value problem for improperly elliptic equation

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The solvability of inhomogeneous third boundary value problem (1) in a unit disk for second order scalar improperly elliptic differential equation (2) with complex coefficients and homogeneous symbol is studied. Such problem has a unique solution in the Sobolev space provided the boundary data belong to the space of functions with exponential decreasing of the Fourier coefficients.

We consider the problem

$$(u'_{\nu_*} - gu)|_{\partial K} = \beta, \tag{1}$$

$$Lu \equiv \left(\sin\varphi_1 \frac{\partial}{\partial x_1} + \cos\varphi_1 \frac{\partial}{\partial x_2}\right)\left(\sin\varphi_2 \frac{\partial}{\partial x_1} + \cos\varphi_2 \frac{\partial}{\partial x_2}\right) u = 0 \tag{2}$$

with complex angles φ_1 and φ_2, where K is a unit disk, $\frac{\partial}{\partial \nu_*}$ is a conormal derivative, $\beta \in H^m_\rho(\partial K)$, $g \in \mathbb{C}\backslash\{0\}$. Besides, we define $H^m_\rho(\partial K)$ as a Sobolev space with weight $\rho = \rho(n)$ to be consisted of the functions

$$\alpha(\tau) = \sum_{n=1}^{\infty} \left(\alpha_n^C \cos n\tau + \alpha_n^S \sin n\tau\right)$$

from $L_2(\partial K)$ such that

$$\sum_{n=1}^{\infty}\big(|\alpha_n^C|^2+|\alpha_n^S|^2\big)\rho^2(n)\big(1+n^2\big)^m<\infty.$$

Additionally, we put

$$\rho=\rho(n)=e^{n(|\mathrm{Im}(\varphi_1+\varphi_2)|-|\mathrm{Im}(\varphi_2-\varphi_1)|)},$$

where $|\mathrm{Im}(\varphi_1+\varphi_2)|-|\mathrm{Im}(\varphi_2-\varphi_1)|>0$ for improperly elliptic equation (2).

Theorem. *Let the angle $\varphi_0=\varphi_2-\varphi_1$ between characteristics be complex and $\beta\in H_\rho^m(\partial K)$. Then there exists the unique solution $u(x)$ of (1), (2), which belongs to $H^{m+3/2}(K)$.*

Integral operators with generalized hypergeometric functions

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Studying of the special functions is prospective and very useful for different branches of science. The continuous development of the mathematical physics, aerodynamics, quantum mechanics, probability theory, astronomy and other has led to the generalization and the creation of new classes of the special functions. Now we consider the confluent generalized hypergeometric function and one of it's application.

Let's obtain compositional relations for the integral operator ${}_{\tau,\beta}\Phi_{\omega;-}^{a,c}$ with right fractional integral I_-^α and derivative D_-^α:

$$(I_-^\alpha\varphi)(x)=\frac{1}{\Gamma(\alpha)}\int_x^\infty\frac{\varphi(t)}{(t-x)^{1-\alpha}}dt,\alpha\in C,Re(\alpha)>0,$$

$$(D_-^\alpha\varphi)(x)=\left(-\frac{d}{dx}\right)^n(I_-^{n-\alpha}\varphi)(x),\alpha\in C,Re(\alpha)>0,n=[Re(\alpha)]+1.$$

Let's introduce the integral operator

$$({}_{\tau,\beta}\Phi_{\omega;-}^{a,c}\varphi)(x)=\int_x^\infty(t-x)^{c-1}{}_1\Phi_1^{\tau,\beta}(a;c;\omega(t-x)^\beta)\varphi(t)dt,$$

where $\varphi\in L(x^{c-1};(1;+\infty))$,

$${}_1\Phi_1^{\tau,\beta}(a;c;z)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)}\times\int_0^1 t^{a-1}(1-t)^{c-a-1}{}_1\Psi_1\left[\begin{array}{c|c}(c;\tau);&\\(c;\beta);&zt^\tau\end{array}\right]dt,$$

here ${}_1\Psi_1(z)$ - special case of the Fox-Wright function [1].

Theorem 1 *If* $\alpha \in C, Re(\alpha) > 0$, $\{a, b, c, \omega\} \subset C, Rec > Reb > 0$,$\{\tau, \beta\} \subset R, \tau > 0, \beta > 0$, $\beta - \tau > 0$, *then for the function* $\varphi \in L(x^{c-1}; (1; +\infty))$*:*

$$I^{\alpha}_{-\tau,\beta}\Phi^{a,c}_{\omega;-} = \frac{\Gamma(c)}{\Gamma(c+\alpha)}{}_{\tau,\beta}\Phi^{a,c+\alpha}_{\omega;-} = {}_{\tau,\beta}\Phi^{a,c}_{\omega;-}I^{\alpha}_{-},$$

$$D^{\alpha}_{-\tau,\beta}\Phi^{a,c}_{\omega;-} = A_{\tau,\beta}\Phi^{a,c-\alpha}_{\omega;-},$$

where $A = \frac{\Gamma(c)\Gamma(c+n-\alpha)}{\Gamma(c+\alpha)\Gamma(c-\alpha)}$, $n = [Re(\alpha)] + 1$.

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Stability of attracting sets for impulsive evolution systems

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As one of the most popular mathematical approaches to the description of evolutionary processes with instantaneous changes, we can mention the theory of impulsive differential equations originated in [1]. An important problem in the theory of impulsive systems of differential equations is a qualitative study of impulsive dynamical systems. In the case of an infinite-dimensional phase space, one of the most effective tools for studying the qualitative behavior of solutions is the theory of global attractors.

However, the transfer of basic concepts of the global attractors theory to impulsive dynamical systems has a fundamental problem – the absence of continuous dependence of solutions on the initial data. This requires a new concept both for the global attractor and for its basic properties (invariance, stability and robustness) [2]. Using the notion of a uniform attractor the results on the existence of uniform attractors for impulsive dynamical systems with infinite number of impulsive points under natural assumptions on systems' parameters were obtained. It turned out that in the case when the trajectories of an impulsive dynamical system G reach the impulsive set M infinitely many times, the uniform attractor Θ can have a non-empty intersection with the impulsive set and be neither invariant nor stable with respect to the impulsive semi-flow. However, it can be proved that

$$D^+(\Theta \setminus M) \subset \overline{\Theta \setminus M}, \tag{1}$$

where $D^+(A) := \bigcup_{x \in A} \{y \mid y = \lim G(t_n, x_n),\ x_n \to x,\ t_n \geq 0\}$.

It is proved that under certain additional restrictions, property (1) remains true for a broad class of impulsive dynamical systems. In particular the obtained results were applied to weakly nonlinear perturbed impulsive system:

$$\begin{cases} \frac{\partial y}{\partial t} = \Delta y + \varepsilon F(y),\ (t, x) \in (0, \infty) \times \Omega, \\ y|_{\partial\Omega} = 0 \end{cases} \tag{2}$$

where $\varepsilon > 0$ is a small parameter and conditions for maps F, I and a set M provide global solvability and dissipativity condition for the impulsive dynamical system in the phase space of the problem.

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Modeling of spin waves propagation in ferromagnetic nanowire

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We develop a continuous medium theory of the dispersion of exchange spin waves in coaxial magnetic nanowire of circular cross section taking into account dipole interaction.

Such system represents an infinite cylinder consisting of coaxial layers with radii R_1 and R_2. The gyromagnetic ratio g, and saturation magnetization M_0 are assumed to be constant throughout the layer. The easy magnetization axis and the applied magnetic field $\mathbf{n}H_0$ are both parallel to the axis of the nanowire ($\mathbf{n}$ is a unit vector oriented along the wire). We assume the absence of magnetization pinning on the external surface of the wire.

Properties of coaxial magnetic nanowire can be described by three main quantities: magnetic field, magnetic induction and magnetization. To describe dynamics of the magnetization $M(r,t)$ in the wire, we will use the Landau-Lifshits equation. Magnetic induction and magnetization may be found from Maxwell equations[3].

$$\frac{\partial M_j}{\partial t} = -g\left[M_j \times H_{\mathrm{eff},j}\right]$$

Here we assume effective field to be equal to $H_{\mathrm{eff},j} = (H_0 + \beta_j M_0)\, n + \alpha_j \Delta M_j + h_{\mathrm{m},j}$where j is number of the layer, α stands for exchange constant. $h_{\mathrm{m,j}} = -\nabla\varphi_j$ describes magnetostatic field.

We use Maxwell equations to determine it $div\,(b_j) = 0$, $rot\,(h_{\mathrm{m,j}}) = 0$.

We consider small amplitude deviations of the magnetization from its ground state $M_j\,(r,t) = nM_{0,j} + m_j\,(r,t)$ and introduce circular variables $\mu_\pm = m_x \pm i m_y$.

After linearizing of Landau- Lifshits equation we have

$$\Omega^2\Delta\varphi_j - (H_j - \alpha_j\Delta)\,(H_j + 4\pi - \alpha_j\Delta)\,\Delta\varphi_j + 4\pi\,(H_j - \alpha_j\Delta)\,\frac{\partial\varphi_j}{\partial z^2} = 0$$

Then we may seek for potential in form of Bessel functions
$\varphi_j = (A_j J_m\,(\kappa_j\rho) + B_j N_m\,(\kappa_j\rho))\, e^{im\phi+ikz}$. Then one may derive relation for magnetization

$$\mu_j = \frac{1}{4\pi}\sum_{n=1}^{3}\left[\begin{array}{c} A_{n,j}\kappa_{n,j}\frac{J_{m+1}(\kappa_{n,j}\rho)}{H_j+\alpha_j(\kappa_{n,j}{}^2+k^2)+\Omega} + \\ +B_{n,j}\kappa_{n,j}\frac{N_{m+1}(\kappa_{n,j}\rho)}{H_j+\alpha_j(\kappa_{n,j}{}^2+k^2)+\Omega}\end{array}\right]\exp\left(i\,(m+1)\,\phi + ikz\right)$$

Applying border conditions on the interface between two layers and external surface of the cylinder we defined implicit relation between frequency Ω and longitudal wave number k in coaxial magnetic nanowire. We suppose thikness of the external layer much smaller then internal. The main result of the given work $\Omega(k)$ dependence was obtained numerically.

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Generalized Poisson's theorem of building first integrals for completely solvable Hamiltonian systems in total differentials

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Consider a Hamiltonian system of equations in total differentials (or Pfaff system) [1, 2]

$$dq_i = \sum_{j=1}^{m} \partial_{p_i} H_j(t,q,p)\, dt_j\,, \quad dp_i = -\sum_{j=1}^{m} \partial_{q_i} H_j(t,q,p)\, dt_j\,, \quad i = 1,\ldots,n, \tag{1}$$

where $q = (q_1,\ldots,q_n) \in \mathbb{R}^n$ and $p = (p_1,\ldots,p_n) \in \mathbb{R}^n$ are the generalized coordinates and momenta, respectively, $t = (t_1,\ldots,t_m) \in \mathbb{R}^m$, and the Hamiltonians $H_j\colon D \to \mathbb{R}$, $j = 1,\ldots,m$, are twice continuously differentiable functions on the domain $D = T \times G$, $T \subset \mathbb{R}^m$, $G \subset \mathbb{R}^{2n}$. If $m = 1$, then we have a canonical Hamiltonian system with n degrees of freedom

$$\frac{dq_i}{dt} = \partial_{p_i} H(t,q,p), \quad \frac{dp_i}{dt} = -\partial_{q_i} H(t,q,p), \quad i = 1,\ldots,n.$$

We assume that the linear differential operators of first order

$$\mathfrak{G}_j(t,q,p) = \partial_{t_j} + \sum_{i=1}^{n} \big(\partial_{p_i} H_j(t,q,p)\, \partial_{q_i} - \partial_{q_i} H_j(t,q,p)\, \partial_{p_i}\big) \quad \forall (t,q,p) \in D, \quad j = 1,\ldots,m,$$

induced by the Hamiltonian system in total differentials (1) are related by the Frobenius conditions [3]. These conditions are represented via Poisson brackets as the system of identities

$$\big[\mathfrak{G}_j(t,q,p), \mathfrak{G}_\xi(t,q,p)\big] = 0 \quad \forall (t,q,p) \in D, \quad j = 1,\ldots,m, \quad \xi = 1,\ldots,m, \tag{2}$$

i.e., the Hamiltonian system in total differentials (1) is completely solvable [4, pp. 15 – 25].

Among the general methods of integration of Hamiltonian systems, the Poisson method is of particular importance. It gives the possibility to find the additional (third) first integral of Hamiltonian system by two known first integrals of Hamiltonian system. And thus, in certain cases, to build an integral basis of Hamiltonian system. Due to this property, the Poisson method is included in almost all monographs and textbooks on analytical mechanics (see, for example, [5, pp. 298 – 306], [6, p. 216]) and formulated in the multidimensional case [1] as the following statement.

Theorem 1 (*the Poisson theorem*). *Suppose twice continuously differentiable functions* $\mathrm{g}_1\colon D' \to \mathbb{R}$ *and* $\mathrm{g}_2\colon D' \to \mathbb{R}$ *are first integrals on the domain* $D' \subset D$ *of the Hamiltonian system* (1). *Then the Poisson bracket*

$$\mathrm{g}_{12}\colon (t,q,p) \to \ [\mathrm{g}_1(t,q,p), \mathrm{g}_2(t,q,p)] \ \ \textit{for all } (t,q,p) \in D'$$

of the functions g_1 *and* g_2 *is also a first integral of the Hamiltonian system* (1).

In his *Lectures on Dynamics* [5, p. 303], C.G.J. Jacobi referred to Poisson's theorem as "one of the most remarkable theorems of the whole of integral calculus. In the particular case when $H = T - U$, it is the fundamental theorem of analytical mechanics. ... After I discovered this theorem I communicated it to the Academies of Berlin and Paris as an entirely new discovery. But I noticed soon after that this theorem had already been discovered and forgotton for 30 years, because one did not appreciate its real meaning, but had only used it as a lemma in a entirely different problem".

Of course, Poisson's theorem does not always supply further first integrals. In some cases the result is trivial, the Poisson bracket being a constant. In other cases the first integral obtained is simply a function of the original integrals. If neither of these two possibilities occurs, however, then the Poisson bracket is a further first integral of the Hamiltonian system (1).

Our aim in this paper is to developed the Poisson theorem (Theorem 1) for integral manifolds of the completely solvable Hamiltonian systems in total differentials (1) ∪ (2). The main result is

Theorem 2 (*the generalized Poisson theorem*). *Suppose* $g_k(t,q,p) = 0,\ g_k \in C^2(D'), k = 1,2,$ *are integral manifolds of the completely solvable Hamiltonian system* (1) ∪ (2) *such that*

$$\mathfrak{G}_j\, g_k(t,q,p) = \Phi_{kj}(t,q,p), \quad \Phi_{kj}(t,q,p)\big|_{g(t,q,p)=0} = 0 \quad \forall (t,q,p) \in D', \quad j = 1,\ldots,m, \quad k = 1,2.$$

Then the function

$$g\colon (t,q,p) \to\ [g_1(t,q,p), g_2(t,q,p)] - \int\limits_{t^0}^{t} \sum_{j=1}^{m} \varphi_j(t)\, dt_j \quad \forall (t,q,p) \in D', \quad t^0 \in T' \subset T,$$

is a first integral of the completely solvable Hamiltonian system (1) ∪ (2) *if and only if the identities hold*

$$\big[g_1(t,q,p),\, \Phi_{2j}(t,q,p)\big] - \big[g_2(t,q,p),\, \Phi_{1j}(t,q,p)\big] = \varphi_j(t) \quad \forall (t,q,p) \in D', \quad \varphi_j \in C(\mathcal{T}'), \quad j = 1,\ldots,m.$$

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Long-time asymptotics for Toda equation with steplike initial data

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We study the asymptotic behavior of the Cauchy problem solutions for the Toda equation

$$\dot{b}(n,t) = 2(a(n,t)^2 - a(n-1,t)^2),\ \dot{a}(n,t) = a(n,t)(b(n+1,t) - b(n,t)), \tag{1}$$

$(n,t) \in \mathbb{Z} \times \mathbb{R}_+$, with steplike initial data

$$a(n,0) \to A,\ b(n,0) \to B,\ n \to -\infty;\ A > 0,\ B \in \mathbb{R},\ a(n,0) \to \frac{1}{2},\ b(n,0) \to 0,\ n \to +\infty. \tag{2}$$

The solution of (1)– (2) is called the rarefaction wave when $B - 2A > 1$, and is called the shock wave when $B + 2A < -1$. We investigate asymptotics for such steplike solutions under an assumption that the ratio $\xi := \frac{n}{t}$ varies slowly, while $n \to \infty, \quad t \to +\infty$. To obtain the asymptotics we apply the nonlinear steepest descend method for the vector oscillatory Riemann-Hilbert problems.

The asymptotics of the Toda rarefaction wave was first studied by this method by Deift et all in [1] for small values of the parameter $\xi := \frac{n}{t} \approx 0$. By means of the same approach, the first and the second terms of the asymptotical expansion with respect to large t were obtained in [2] in all principal regions of (n,t) half- plane, yet without rigorous justification. In [3] these asymptotics are fully justified.

The case of the Toda shock wave is more complicated for investigation. There are five regions on (n,t) half-plane with different asymptotical behavior of the solution. Applying the Lax-Levermore approach, at the physical level of rigor Venakides, Deift and Oba in [4] showed that in the central region the solution is close to the periodic solution of the Toda lattice of period 2. We obtain and rigorously justify the asymptotics of the Toda shock wave in the elliptic wave region, where the solution of (1)–(2) turns out to be close to a finite gap (two- band) quasi-periodic solution of the Toda lattice for each fixed ξ. This is the result of joint work with I.Egorova, J. Michor and G. Teschl.

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About solution of the boundary problem on the eigenwaves of a comb-shaped slot resonator

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A technique of the boundary value problem solving for the eigenfunction and eigenfrequencies of a comb slot resonator is proposed. Resonators of this type are used to design of broadband rejection filters, in particular, high-harmonic filters. [1,2,3].

On practice the two boundary value problems, "electric" and "magnetic", with two different conditions in the symmetry plane $z = 0$ and the condition of the magnetic wall in the plane $x = 0$ are solved. The condition of the magnetic wall (m.w.) corresponds to zero derivative of the basis function in this plane, and the condition of the electrical wall (e.w.) corresponds to zero of the function on this boundary. Functions $T_{hy,k}(x, z)$ are orthogonal and satisfy to the wave equation:

$$\Delta T_{hy,k} + k^2{}_{hc,k} T_{hy,k} = 0 \tag{1}$$

and boundary conditions $\frac{dT_{hy}}{dn} = 0$ on the perfect metal wall. As an example, consider the solution of the electric boundary value problem. The function $T_{hy,k}(x, z)$ in partial domains with the conditions of the magnetic wall a $x = 0$ and the electric wall at $z = 0$

is written as an expansion in Fourier series with unknown coefficients:

$$T_{h_1}(x,z)=\sum_{k=0} A_{h_1k}\sqrt{\frac{4-2\cdot\delta_{k_0}}{s_1}}\cdot\cos\frac{2\pi k}{s_1}z\cdot\frac{\cos k_{x_1k(l_1-x)}}{k_{x_1k}\sin k_{x_1k}d_1},$$

$\frac{s}{2}\le x\le l_1, 0\le z\le\frac{s_1}{2}, d_1=l_1-\frac{s}{2}$;

$$T_{h_2}(x,z)=\sum_{k=0} A_{h_2k}\sqrt{\frac{2-\delta_{k_0}}{s_2}}\cdot\cos\frac{\pi k}{s_2}(z-g-\frac{s_1}{2})\cdot\frac{\cos k_{x2k}(l_2-x)}{k_{x2k}\sin k_{x2k}d_2},$$

$\frac{s}{2}\le x\le l_2, g+\frac{s_1}{2}\le z\le b_s, d_2=l_2-\frac{s}{2}$;

$$T_{h_3}(x,z)=\sum_{k=1}\frac{2}{\sqrt{s}}\cdot\sin\frac{\pi(2k-1)}{s}x\cdot[A_{h_{31},k}\frac{\cos k_{z_1k}(z-z_0)}{k_{z_1k}\sin k_{z_1k}\frac{g}{2}}+A_{h_{32},k}\frac{\sin k_{z_1k}(z-z_0)}{k_{z_1k}\cos k_{z_1k}\frac{g}{2}}],$$

$\frac{s_1}{2}\le z\le g+\frac{s_1}{2}, 0\le x\le\frac{s}{2}, z_0=\frac{s_1}{2}+\frac{g}{2}$;

$$T_{h_4}(x,z)=\sum_{n=1} B_{h_ln}\frac{2}{\sqrt{s}}\cdot\sin\frac{\pi(2n-1)}{s}x\cdot\frac{\cos k_{z_1n}z}{k_{z_1n}\sin k_{z_1n}\frac{s_1}{2}}+$$

$$\sum_{k=0} B_{h_2n}\frac{4-2\cdot\delta_{n_0}}{\sqrt{s_1}}\cos\frac{2\pi n}{s_1}z\cdot\frac{\sin k_{z_1n}x}{k_{x_1n}\cos(k_{x_1n}\frac{s}{2})},$$

$0\le x\le\frac{s}{2}, 0\le z\le\frac{s_1}{2}$.

$$T_{h_5}(x,z)=\sum_{k=1} C_{h_lk}\frac{2}{\sqrt{s}}\sin\frac{\pi(2n-1)}{s}x\cdot\frac{\cos k_{z_1k}(b_s-z)}{k_{z_1k}\sin(k_{z_1k}s_2)}+$$

$$\sum_{k=0} C_{h_2k}\sqrt{\frac{2-\delta_{n_0}}{s_1}}\cos\frac{\pi k}{s_2}(z-g-\frac{s_1}{2})\frac{\sin k_{x_2k}x}{k_{x_2k}\cos(k_{x_2k}\cdot\frac{s}{2})},$$

$0\le x\le\frac{s}{2}, \frac{s_1}{2}+g\le z\le b_s$, when ${k_{z_1n}}^2={k_{hc}}^2-(\frac{\pi(2n-1)}{s})^2$,${k_{x_1n}}^2={k_{hc}}^2-(\frac{2\pi n}{s_1})^2$, ${k_{x_2n}}^2={k_{hc}}^2-(\frac{\pi n}{s_2})^2$.

From continuity conditions function and its derivative at the boundaries by the partial regions the system of linear algebraic equations (SLAE) is obtained in the form:

$$\sum_{k=0} B_{h_2k}(F_{h_1,n}\delta_{nk}-\sum_{m=1}\frac{F_{4m}}{D_m}S_{1nm}S_{1km})+\sum_{k=0} C_{h_2k}\sum_{m=1}\frac{F_{gm}}{D_m}S_{1nm}S_{3km}=0,$$

$$\sum_{k=0} B_{h_2k}\sum_{m=1}\frac{F_{gm}}{D_m}S_{3nm}S_{1km}+\sum_{k=0} C_{h_2k}(F_{2n}\delta_{nk}-\sum_{m=1}\frac{F_{3m}}{D_m}S_{3km}S_{3nm})=0, n=0,1,2....$$

or in block matrix form:

$$\begin{bmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{pmatrix} B_{h_2}\\ C_{h_2}\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}. \tag{2}$$

The matrices A_{11} и A_{22} in each block of SLAE (2) are symmetric, and the side matrices are related by $A_{21} = A^T{}_{12}$. Characteristic equation for determining eigenvalue $k^2{}_{hc}$ (spectrum of eigenvalues) is a transcendental equation, where the determinant Δ of the matrix A is zero ($|A| = 0$). From the solution of SLAE, are found all coefficients of expansion with the accuracy of some of the multiplier. In the case when all four matrices$A_{11}, A_{12}, A_{21}, A_{22}$ are square (of the same order M) it is convenient to use Schur's formulas, which reduce the calculation of the $2M^{th}$ order determinant to the calculation of the M^{th} order determinant. When $|A_{11}| \neq 0$ the determinant is

$$\Delta = |A_{11}||A_{22} - A_{21}{A_{11}}^{-1}A_{12}| \approx |A_{11}||A_{22}|,$$

and to find the first series of roots of the characteristic equation, it suffices to equal to zero the determinant $\Delta = |A_{11}||A_{22} - A_{21}{A_{11}}^{-1}A_{12}| = 0$. Similarly, when $|A_{22}| \neq 0$

$$\Delta = |A_{22} - A_{12}{A_{22}}^{-1}A_{21}||A_{22}| \approx |A_{11}||A_{22}|,$$

and to find the roots it is enough to equate the determinant to zero $|A_{22} - A_{12}{A_{22}}^{-1}A_{21}||A_{22}| = 0$. The multiplier factor is obtained from the normalization condition:

$$\int_{S_t} \nabla T_{h,k} \nabla T_{h,l} dS = \delta_{kl} = k^2{}_{ch} \int_{S_t} T^2{}_{h,l} dS.$$

Table 1 shows the values of the first three roots of the characteristic equation (two series of solutions) for a comb slot resonator with parameters (in mm): $s = 0.6$, $s_1 = s_2 = 0.4$, Table 1 obtained by numerical solution of the boundary value problem by series reducing to $M = 5$, corresponding values of critical frequencies in GHz are given in brackets.As expected, the first frequencies correspond to the eigenfrequencies of narrow ($\frac{s_i}{2l_i} > 1, i = 1, 2$) rectangular slot resonators (two series of solutions) $\chi_{i,n_0} = \frac{\pi(2n-1)}{2l_i}, i = 1, 2, n = 1, 2, ...N$.

Table 1 - Eigenvalues of comb waveguide (by condition e.w. at $z = 0$)

$\chi_1(mm^{-1}, GHz)$	$\chi_2(mm^{-1}, GHz)$
0.17379(8.297)	0.3119(14.895)
0.5213(24.892)	0.9357(44.677)
1.2162(58.069)	1.5588(74.429)

Similar results are obtained when solving the "magnetic" boundary value problem. In this case, the eigenfrequencies correspond to only one series $\chi_{2,n_0} = \frac{\pi(2n-1)}{2l_2}$ (the field in the first slot resonator corresponds to the reactive mode).

The proposed solution technique can also be used to calculate the eigenfunctions of comb resonators with a large number of electromagnetically connected slots.

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Long-time asymptotics for the integrable nonlocal nonlinear Schrödinger equation

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We consider the Cauchy problem for the so-called integrable nonlocal nonlinear Schrödinger equation

$$iq_t(x,t) + q_{xx}(x,t) + 2\sigma q^2(x,t)\bar{q}(-x,t) = 0,\ \sigma = \pm 1, \quad x \in (-\infty, \infty),\ t > 0, \tag{1}$$

$$q(x,0) = q_0(x), \quad x \in (-\infty, \infty), \tag{2}$$

in the class of rapidly decaying to 0 functions. The nonlocal nonlinear Schrödinger equation (1) was introduced by M. Ablowitz and Z. Musslimani in [1] and has attracted much attention in recent years due to its interesting physical and mathematical properties. The main aim of our work is to obtain the long-time asymptotics of the solution of the Cauchy problem (1), (2). By applying the Inverse Scattering Transform method we reduce (1) to the matrix Riemann-Hilbert factorization problem (RHP) with the (oscillatory) jump matrix determined in terms of the initial data (2). The long-time asymptotics solutions of the RHP can be obtained by using the Deift and Zhou nonlinear steepest decent method [2] under certain assumptions on the index of the associated scalar Riemann-Hilbert problem. The main difference in the analysis of this problem comparing with the corresponding problem for the conventional (local) nonlinear Schrödinger equation is lack of some symmetries of the spectral functions associated to the initial data $q_0(x)$. Consequently, the decay of the principal term in asymptotic formula turns to depend, in general, on the direction $\frac{x}{t} = const$ [3]. This is in sharp contrast with the case of the local nonlinear Schrödinger equation, where the order of decay is always $t^{-1/2}$, independently of the direction in the (x,t) plane.

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Modeling and homogenization of hydrodynamics processes with the vanishing viscosity

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Initial boundary value problems for nonstationary linearized equations of hydrodynamics, Stokes and Navier–Stokes equations with the vanishing viscosity and periodic data rapidly oscillating with respect to the spatial variables, will be discussed. The problems are stated in bounded domains that are three-dimensional, for example. The period of data oscillations is specified by a positive small parameter ε and a viscosity coefficient ν in equations of the problems can be also considered as a positive parameter. We present estimates of solutions of the problems, which are dependent on relations of certain powers of the parameters ε and ν. In general case, the presented estimates for velocity fields are actual whenever the viscosity coefficient ν is not too small in comparison with ε^2. If the condition is fulfilled, then the relevant solutions are small asymptotically in an energy norm and it characterizes a "smoothing" property for these solutions. In the case, when the viscosity coefficient has order ε^2, the suitable estimates are derived under assumption that a nonlinearity in equations of the problems is "small" sufficiently. If the condition is fulfilled, then an asymptotics for velocity fields can contain rapidly oscillating terms for the solutions of Stokes and Navier–Stokes equations.

Thus, homogenized (limit) equations whose solutions determine approximations (leading terms of the asymptotics) of the solutions of the equations under consideration and estimate the accuracy of the approximations will be obtained. These approximations and estimates shed light on the following interesting property of the solutions of the equations. When the viscosity is not too small, the approximations contain no rapidly oscillating terms, and the equations under consideration asymptotically smooth the rapid oscillations of the data; thus, the homogenized equations are asymptotically parabolic. If the viscosity is very small, the approximations can contain rapidly oscillating terms with zero means, and the homogenized equations are asymptotically hyperbolic.

The homogenization of some cases of nonstationary linearized equations of hydrodynamics, Stokes and Navier–Stokes equations with periodic rapidly oscillating "forces" were considered in [1] and [2]. In particular, the presented approximations and results are applicable to some Kolmogorov flows in classification of [3]. Some generalizations of these results are given in [4]. Asymptotic and homogenization methods of [1] and [4] are used to derive the presented approximations and results.

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Homogenized models with memory effects for composites

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Diffusion unsteady processes in the composite material consisting of several materials with widely different properties will be discussed. The conductivity and density of one of the materials is assumed to be considerably lower than the conductivity of the other materials. The diffusion and wave processes under study are governed by parabolic and hyperbolic equations with coefficients depending on two small positive parameters ε and σ. The first microscale parameter ε determines the period of the coefficients in these equations, which corresponds to the assumption that the composite material under study has a periodic structure with period ε. The inverse of the second parameter σ characterizes the scatter of the conductivities in these equations, which corresponds to the assumption that one of the materials has a very low conductivity as compared to the others.

Homogenized equations and systems of equations whose solutions approximate the solutions to the original equations and estimate the accuracy of such approximations will be presented. Under certain assumptions on the geometry of the periodic distribution of the constituting materials in space, the homogenized equations form a system of equations with convolution coupled through exchange coefficients that is a multiphase model. The model coefficients characterize dynamic diffusion exchange between the materials viewed as components of the composite material under consideration and are involved in the homogenized equations via convolution operators with respect to the time variable. Such terms with time convolution in equations has a well-known mechanical interpretation corresponding to the memory effect arising in the homogenized medium. Moreover, multiphase models or multiphase flows equations are usually derived by using the concept of multivelocity continuum and the assumption of interpenetrating motion of the components. In a sense, this concept means that several materials are simultaneously present at each spatial point under consideration. Here we discuss another approach to multiphase models with memory effects that is arisen from homogenization theory.

The homogenized coupled equations and accuracy estimate admit also a natural interpretation of the multiphase flows equations for the particular diffusion models. Each equation of such homogenized system characterizes diffusion in the domain occupied by a particular (well conducting) material, and diffusion exchange between these materials is determined by the exchange coefficients coupling the equations. Some cases of the homogenized multiphase models with memory effects were derived in [1] and [2].

Special case of the models is a well-known model of parallel flows in mechanics of porous media [3], where additional details and bibliography may be found. The asymptotic and homogenization methods are actual also for spectral problems according to [4], where generalizations of the methods are given.

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About one class of continual approximate solutions

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The kinetic equation Boltzmann is the main instrument to study the complicated phenomena in the multiple-particle systems, in particular, rarefied gas. This kinetic integro-differential equation for the model of hard spheres has a form [1, 2]:

$$D(f) = Q(f, f).$$

We will consider the continual distribution [3]:

$$f = \int\limits_{\mathbb{R}^3} \varphi(t, x, u) M(v, u, x) du,$$

which contains the local Maxwellian of special form describing the screw-shaped stationary equilibrium states of a gas (in short-screws or spirals) [4]. They have the form:

$$M(v, u, x) = \rho_0 e^{\beta\omega^2 r^2} \left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} e^{-\beta(v-u-[\omega\times x])^2}. \tag{1}$$

Physically, distribution (1) corresponds to the situation when the gas has an inverse temperature $\beta = \frac{1}{2T}$, where $T = \frac{1}{3\rho} \int\limits_{\mathbb{R}^3} (v-u)^2 f dv$ and rotates in whole as a solid body with the angular velocity $\omega \in R^3$ around its axis on which the point $x_0 \in R^3$ lies,

$$x_0 = \frac{[\omega \times u]}{\omega^2}.$$

The square of this distance from the axis of rotation is

$$r^2 = \frac{1}{\omega^2} [\omega \times (x - x_0)]^2$$

and the density of the gas has the form:

$$\rho = \rho_0 e^{\beta\omega^2 r^2}.$$

Here ρ_0 is the density of the axis, that is $r = 0$, $u \in R^3$ is the arbitrary parameter (linear mass velocity for x), for which $x || \omega$ and $u + [\omega \times x]$ is the mass velocity in the arbitrary point x. The distribution (1) gives not only a rotation, but also a translational movement along the axis with the linear velocity

$$\frac{(\omega, u)}{\omega^2} \omega.$$

Thus, it really describes a spiral movement of the gas in general, moreover, this distribution is stationary (independent of t), but inhomogeneous.

The purpose is to find such a form of the function $\varphi(t,x,u)$ and such a behavior of all hydrodynamical parameters so that the uniform-integral remainder [3, 4]

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int_{\mathbb{R}^3} |D(f) - Q(f,f)|dv$$

or its modification "with a weight":

$$\widetilde{\Delta} = \sup_{(t,x)\in\mathbb{R}^4} \frac{1}{1+|t|} \int_{\mathbb{R}^3} |D(f) - Q(f,f)|dv$$

tends to zero.

Also some sufficient conditions to minimization of remainder Δ and $\widetilde{\Delta}$ are found. The obtained results are new and may be used with the study of evolution of screw and whirlwind streams.

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Removability result for anisotropic parabolic equations

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In the cylinder domain $\Omega\times(0,T)$ we consider a class of quasilinear parabolic equations model of which are

$$u_t - \sum_{i=1}^{n} \left(u^{m_i-1}u_{x_i}\right)_{x_i} + f(u) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} - \sum_{i=1}^{n} \left(u^{m_i-1}u_{x_i}\right)_{x_i} + \sum_{i=1}^{n} |u_{x_i}|^{q_i} = 0, \tag{2}$$

where Ω is a bounded domain in R^n, $n \geq 2$, $t \in (0,T)$, $0 < T < +\infty$, $0 \in \Omega$. We focus on the nonnegative solutions which satisfy the initial condition

$$u(x,0) = 0, \;\; x \in \Omega \setminus \{0\}. \tag{3}$$

Suppose that

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq \ldots \leq m_n < m + \frac{2}{n}, m = \frac{1}{n}\sum_{i=1}^{n} m_i, \tag{4}$$

$$\frac{2+nm}{1+n} \leq q < 2, \quad \max_{0\leq i \leq n} q_i < q\left(1+\frac{1}{n}\right), \quad \frac{1}{q} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{q_i}. \tag{5}$$

The main purpose is to obtain sufficient condition for removability of the singularity for solutions of equations (1), (2). Let us formulate result for equation (1).

Theorem 1 [1] *Let the condition (4) be fulfilled and u be a weak solution to the problem (1), (3). Assume also that $f(u) = u^q$ and*

$$q \geq m + \frac{2}{n},$$

then the singularity at the point $\{(0,0)\}$ is removable.

The proof of removability result is based on the new a priori estimates of "large" type solutions. In particular, we obtain the Keller-Osserman type estimate of the solution to the equation (1). Let $(x^{(0)}, t^{(0)}) \in \Omega_T$, for any $\tau, \theta_1, \theta_2, ..., \theta_n > 0$, $\theta = (\theta_1, ..., \theta_n)$ we define $Q_{\theta,\tau}(x^{(0)}, t^{(0)}) := \{(x,t) : |t - t^{(0)}| < \tau, |x_i - x_i^{(0)}| < \theta_i, i = \overline{1,n}\}$ and set $M(\theta,\tau) := \sup\limits_{Q_{\theta,\tau}(x^{(0)},t^{(0)})} u$, $F(\theta,\tau) := \sup\limits_{Q_{\theta,\tau}(x^{(0)},t^{(0)})} F(u)$, $F(u) = \int_0^u s^{m^- -1} f(s)ds$, $m^+ = max(m_n, 1)$, $m^- = min(m_1, 1)$.

Theorem 2 [1] *Let the condition (4) be fulfilled and u be a weak solution to equation (1), assume also that $f \in C^1(R^1_+)$ and $f'(u) \geq 0$. Let $(x^{(0)}, t^{(0)}) \in \Omega_T$, fix $\sigma \in (0,1)$, and let $Q_{8\theta,8\tau}(x^{(0)}, t^{(0)}) \subset \Omega_T$. Set $\rho = \begin{cases} \theta_n, \text{if } m_n > 1, \\ \tau^{\frac{1}{2}}, \text{if } m_n < 1, \end{cases}$, then there exist positive number c_1, c_2 depending only on $n, \nu_1, \nu_2, m_1, ..., m_n$ such that either*

$$u(x^{(0)}, t^{(0)}) \leq \left(\frac{\theta_n^2}{\tau}\right)^{\frac{1}{m_n-1}} + \sum_{i=1}^{n-1}\left(\frac{\rho}{\theta_i}\right)^{\frac{2}{m^+-m_i}},$$

or

$$(M(\sigma\theta,\sigma\tau))^{1-m^- + \frac{n(m-m^-)}{2}} F(M(\sigma\theta,\sigma\tau)) \leq$$
$$\leq c_1(1-\sigma)^{-\gamma}\rho^{-2}(M(\theta,\tau))^{m^+ + 1 + \frac{n(m-m^-)}{2}}$$

holds true.

We also have, in particular, if

$$F(\varepsilon u) \leq \varepsilon^{m^+ + m^- + \beta} F(u), \; \beta > 0,$$

then

$$F(M(\theta,\tau)) \le c_2(1-\sigma)^{-\gamma} M^{m^+ + m^-}(\theta,\tau)\rho^{-2}.$$

The following theorems are the removability result and Keller-Osserman estimates for the equation (2)

Theorem 3. [2] *Let u be a weak solution to the problem (2), (3). Let that the conditions (4),(5) be fulfilled, and assume also that if $q = \frac{2+nm}{1+n}$ then $q_i = \frac{2+nm}{1+n+\frac{n}{2}(m-m_i)}$, $i = \overline{1,n}$. Then the singularity at the point $\{(0,0)\}$ is removable.*

Theorem 4. [2] *Let that the conditions (4),(5) be fulfilled. Then there exists a positive constant c depending only on $\nu_1, \nu_2, n, m_1, \dots, m_n, q_1, \dots, q_n$ such that the following inequality holds*

$$|u(x,t)| \le c \left(\sum_{i=1}^{n} |x_i|^{\frac{2}{(2-m)q+(q-2)m_i}} + t^{\frac{1}{q(1-m)+2(q-1)}} \right)^{q-2},$$

for $(x,t) \in \Omega_T \setminus \{(0,0)\}$.

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Quasiconformal mappings. Global and almost-global geodesic half geodesic parameterizations of the surfaces

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The differential equation for the topological mapping transforming first quadratic form of Liouville surface parametrized by isothermal coordinates into that of half-geodesic one is deduced.

The equation is quasilinear conjugated Beltrami equation. In order to prove existence of diffeomorphism leading to half-geodesic parameterization of Liouville surface the method of successive approximations is used.

The diffeomorphisms of the said equation are used in order to prove the possibility of immersion of the first quadratic form $(\phi(u) + \psi(v))\left(du^2 + dv^2\right)$ into R^3 by Liouville surface.

For general C^1 – surfaces Beltrami equation is non-linear one and admits degeneration at the points where Jacobian of diffeomorphism turns to be equal to zero or infinity.

The method of successive approximations is now divided into two stages: at the first stage the sequence of $K(m)$-quasiconformal is constructed with $K(m)$ possibly tending to infinity and on the second stage this sequence is used in order to prove the existence of homeomorphism, the solution of the said non-linear Beltrami equation.

Fourier Problem for Weakly Nonlinear Evolution Inclusions with Functionals

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Let $S := (-\infty, 0]$, V and H be separable Hilbert spaces with the scalar products $(\cdot,\cdot)_V$, $(\cdot,\cdot)$ and norms $\|\cdot\|$, $|\cdot|$, respectively. Suppose that $V \subset H$ with dense, continuous and compact injection. Denote by V' and H' the dual spaces to V and H, respectively. We suppose that the space H' is a subspace of V'. Identifying the spaces H and H' by the Riesz-Fréchet representation theorem, we obtain dense and continuous embeddings

$$V \subset H \subset V'.$$

We introduce some function spaces. Let X be an arbitrary Hilbert space with the scalar product $(\cdot,\cdot)_X$ and the norm $\|\cdot\|_X$. Denote by $L^2_{\mathrm{loc}}(S;X)$ the linear space of measurable functions defined on S with values in X, whose restrictions to any segment $[t_1, t_2] \subset S$ belong to $L^2(t_1, t_2; X)$. Let $\nu \in \mathbb{R}$. Put by definition

$$L^2_\nu(S;X) := \Big\{ f \in L^2_{\mathrm{loc}}(S;X) \;\Big|\; \int_S e^{2\nu t}\|f(t)\|_X^2\,dt < \infty \Big\}.$$

This space is a Hilbert space with the scalar product and the corresponding norm

$$(f,g)_{L^2_\nu(S;X)} = \int_S e^{2\nu t}(f(t),g(t))_X\,dt, \quad \|f\|_{L^2_\nu(S;X)} := \Big(\int_S e^{2\nu t}\|f(t)\|_X^2\,dt \Big)^{1/2}.$$

Also we introduce the spaces

$$L^\infty_\nu(S;X) := \{ f \in L^\infty_{\mathrm{loc}}(S;X) \mid \operatorname*{ess\,sup}_{t\in S} \left[e^{\nu t}\|f(t)\|_X \right] < \infty \},$$

$$H^1_\nu(S;H) := \{ w \in L^2_\nu(S;H) \,\big|\, w' \in L^2_\nu(S;H) \}, \quad \nu \in \mathbb{R}.$$

Let $\Phi : V \to \mathbb{R}_\infty := (-\infty, +\infty]$ be a proper functional, i.e., $\mathrm{dom}(\Phi) := \{ v \in V : \Phi(v) < +\infty \} \neq \emptyset$, which satisfies the conditions:

($\mathcal{A}_1$) $\quad \Phi\big(\alpha v+(1-\alpha)w\big)\le \alpha\Phi(v)+(1-\alpha)\Phi(w) \quad \forall\, v,w\in V,\ \forall\,\alpha\in[0,1],$

i.e., the functional Φ is *convex*,

($\mathcal{A}_2$) $\quad v_k \underset{k\to\infty}{\longrightarrow} v$ in $V \quad \Longrightarrow \quad \varliminf_{k\to\infty} \Phi(v_k)\ge\Phi(v),$

i.e., the functional Φ is *lower semicontinuous*.

Recall that the *subdifferential* of functional Φ is a mapping $\partial\Phi: V\to 2^{V'}$, defined as follows

$$\partial\Phi(v):=\{v^*\in V' \mid \Phi(w)\ge\Phi(v)+(v^*,w-v) \quad \forall\, w\in V\}, \quad v\in V,$$

and the *domain* of the subdifferential $\partial\Phi$ is the set $D(\partial\Phi):=\{v\in V\,|\,\partial\Phi(v)\neq\emptyset\}$. We identify the subdifferential $\partial\Phi$ with its graph, assuming that $[v,v^*]\in\partial\Phi$ if and only if $v^*\in\partial\Phi(v)$, i.e., $\partial\Phi=\{[v,v^*]\mid v\in D(\partial\Phi),\ v^*\in\partial\Phi(v)\}$.

Let $B(t,\cdot): H\to H$, $t\in S$, be a family of operators which satisfies the condition:

($\mathcal{B}$) for any $v\in H$ the mapping $B(\cdot,v): S\to S$ is measurable, and there exists a constant $L\ge 0$ such that following inequality holds

$$|B(t,v_1)-B(t,v_2)|\le L|v_1-v_2|$$

for a.e. $t\in S$, and for all $v_1,v_2\in H$; in addition, $B(t,0)=0$ for a.e. $t\in S$.

Consider the evolutionary variational inequality

$$u'(t)+\partial\Phi\big(u(t)\big)+B(t,u(t))\ni f(t), \quad t\in S, \tag{1}$$

where $f: S\to V'$ is a given measurable function and $u: S\to V$ is an unknown function.

Definition 1. Let conditions ($\mathcal{A}_1$), ($\mathcal{A}_2$), ($\mathcal{B}$) hold, and $f\in L^2_{\rm loc}(S;V')$. The *solution* of variational inequality (1) is called a function $u: S\to V$ that satisfies the following conditions:

1) $u\in L^2_{\rm loc}(S;V)$, $u'\in L^2_{\rm loc}(S;V')$;

2) $u(t)\in D(\partial\Phi)$ for a.e. $t\in S$;

3) there exists a function $g\in L^2_{\rm loc}(S;V')$ such that, for a.e. $t\in S$, $g(t)\in\partial\Phi\big(u(t)\big)$ and

$$u'(t)+g(t)+B(t,u(t))=f(t) \quad \text{in } V'.$$

We will consider the problem of finding a solution of variational inequality (1) (for given Φ,B,f) satisfying the condition

$$\lim_{t\to-\infty} e^{\gamma t}|u(t)|=0,$$

for given $\gamma\in\mathbb{R}$.

This problem is called the problem $\mathbf{P}(\Phi,B,f,\gamma)$, and the function u is called its solution.

Additionally, assume that the following conditions hold:

($\mathcal{A}_3$) there exists a constant $K_1 > 0$ such that

$$(v_1^* - v_2^*, v_1 - v_2) \geq K_1 |v_1 - v_2|^2 \quad \forall\, [v_1, v_1^*],\ [v_2, v_2^*] \in \partial\Phi;$$

($\mathcal{A}_4$) there exists a constant $K_2 > 0$ such that

$$\Phi(v) \geq K_2 \|v\|^2 \quad \forall\, v \in \mathrm{dom}(\Phi);$$

moreover, $\Phi(0) = 0$.

The main results are the theorems of uniqueness and existence of the problem $\mathbf{P}(\Phi, B, f, \gamma)$.

Theorem 1. Let conditions ($\mathcal{A}_1$) – ($\mathcal{A}_3$), ($\mathcal{B}$) hold, and $\gamma \in \mathbb{R}$ is such that

$$\gamma < K_1 - L. \tag{2}$$

Then the problem $\mathbf{P}(\Phi, B, f, \gamma)$ has at most one solution.

Theorem 2. Let conditions ($\mathcal{A}_1$) – ($\mathcal{A}_4$), ($\mathcal{B}$) hold, and

($\mathcal{F}$) $\quad f \in L^2_\gamma(S; H)$,

where $\gamma \in \mathbb{R}$ satisfies inequality (2). Then the problem $\mathbf{P}(\Phi, B, f, \gamma)$ has a unique solution, it belongs to the space $L^\infty_\gamma(S; V) \cap L^2_\gamma(S; V) \cap H^1_\gamma(S; H)$, and satisfies the estimate:

$$e^{2\gamma\sigma}\|u(\sigma)\|^2 + \int\limits_{-\infty}^{\sigma} e^{2\gamma t}\|u(t)\|^2\,dt + \int\limits_{-\infty}^{\sigma} e^{2\gamma t}|u'(t)|^2\,dt$$

$$+ \int\limits_{-\infty}^{\sigma} e^{2\gamma t}\Phi(u(t))dt \leq C \int_{-\infty}^{\sigma} e^{2\gamma t}|f(t)|^2\,dt, \quad \sigma \in S,$$

where C is a positive constant depending on K_1, K_2, L, and γ only.

Evolutionary variational inequalities with operators type Volterra

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Let $T > 0$ be an arbitrary fixed number, V be a separable reflexive Banach space with the norm $\|\cdot\|$, H be the Hilbert space with the scalar product $(\cdot,\cdot)$ and the norm $|\cdot|$. Suppose that $V \subset H$ is dense, continuous and compact embedding. Let V' and H' be the dual spaces to V and H, respectively. We suppose, that the space H' is a subspace of V'.

Identifying by the Riesz-Fréchet representation theorem the spaces H and H', we obtain dense and continuous embeddings

$$V \subset H \subset V' .$$

Let us define the spaces

$$H^1(0,T;H) := \{w \in L^2(0,T;H) \bigm| w' \in L^2(0,T;H)\},$$

$$W_p^1(0,T;V) := \{w \in L^p(0,T;V) \bigm| w' \in L^{p'}(0,T;V')\}, \qquad p > 1,\ \ p' := p/(p-1).$$

In this talk, we deal with the existence and the uniqueness of solutions to the problem for evolutionary variational inequalities (subdifferential inclusions) with operators type Volterra.

Let $\Phi : V \to (-\infty, +\infty]$ be a proper functional, i.e., $\mathrm{dom}(\Phi) := \{v \in V \mid \Phi(v) < +\infty\} \neq \varnothing$, which satisfies the following conditions:

$(\mathcal{A}_1):\quad \Phi\bigl(\alpha v + (1-\alpha)w\bigr) \leqslant \alpha\Phi(v) + (1-\alpha)\Phi(w) \quad \forall\, v, w \in V,\ \forall\, \alpha \in [0,1],$

i.e., the functional Φ is *convex*,

$(\mathcal{A}_2):\quad v_k \underset{k\to\infty}{\longrightarrow} v \ \text{ in } \ V \quad \Longrightarrow \quad \underset{k\to\infty}{\underline{\lim}}\, \Phi(v_k) \geqslant \Phi(v),$

i.e., the functional Φ is *lower semicontinuous*.

Recall that the *subdifferential* of functional Φ is a mapping $\partial\Phi : V \to 2^{V'}$, defined as follows

$$\partial\Phi(v) := \{v^* \in V' \mid \Phi(w) \geqslant \Phi(v) + (v^*, w - v) \quad \forall\, w \in V\}, \quad v \in V,$$

and the *domain* of the subdifferential $\partial\Phi$ is the set $D(\partial\Phi) := \{v \in V \mid \partial\Phi(v) \neq \varnothing\}$.

Additionally, assume that the following condition hold:

$(\mathcal{A}_3):$ there exist constants $p \geqslant 2,\ K > 0$ such that

$$\Phi(v) \geqslant K\|v\|^p \quad \forall\, v \in \mathrm{dom}(\Phi);$$

moreover, $\Phi(0) = 0$.

Let $\mathcal{B} : L^2(0,T;H) \to L^2(0,T;H)$ be an operator which satisfies the condition

$(\mathcal{A}_4):$ there exists a constant $L \geqslant 0$ such that, for a.e. $t \in (0,T)$ and for any $w_1, w_2 \in L^2(0,T;H)$, the following inequality holds:

$$|\mathcal{B}(w_1)(t) - \mathcal{B}(w_2)(t)| \leqslant L \int_0^t |w_1(s) - w_2(s)|\, ds;$$

moreover, $\mathcal{B}(0) = 0$.

Operator $\mathcal{B}$ is called an operator *type Volterra.*

Let us consider the ***evolutionary variational inequality (subdifferential inclusion)***

$$u'(t) + \partial\Phi\big(u(t)\big) + \mathcal{B}(u)(t) \ni f(t), \quad t \in (0,T), \tag{1}$$

where $f : (0,T) \to V'$ is a given measurable function, $u : (0,T) \to V$ is an unknown function.

Definition. *Let conditions* $(\mathcal{A}_1)$ – $(\mathcal{A}_4)$ *hold, and* $f \in L^{p'}(0,T;V')$*, where* $p' = p/(p-1)$*. The* solution *of variational inequality (1) is called a function* u *that satisfies the following conditions*

1) $u \in W^1_p(0,T;V)$ *(then* $u \in C([0,T];H)$*);*

2) $u(t) \in D(\partial\Phi)$ *for a.e.* $t \in (0,T)$*;*

3) *there exists a function* $g \in L^{p'}(0,T;V')$ *such that, for a.e.* $t \in (0,T)$*, we have* $g(t) \in \partial\Phi\big(u(t)\big)$ *and*

$$u'(t) + g(t) + \mathcal{B}(u)(t) = f(t) \quad \text{in} \quad V'.$$

We consider the ***problem*** of finding a solution u of the variational inequality (1) that satisfies the following condition

$$u(0) = u_0,$$

where $u_0 \in H$ is given.

It is called the problem $\mathbf{P}(\Phi, \mathcal{B}, f, u_0)$, and the function u is called its solution.

Theorem 1 *(uniqueness of the solution). Let conditions* $(\mathcal{A}_1)$ – $(\mathcal{A}_4)$ *hold, and* $f \in L^{p'}(0,T;V')$*,* $u_0 \in H$*. Then the problem* $\boldsymbol{P}(\Phi, \mathcal{B}, f, u_0)$ *has no more than one solution.*

Theorem 2 *(existence of the solution). Let conditions* $(\mathcal{A}_1)$ – $(\mathcal{A}_4)$ *hold, and* $f \in L^2(0,T;H)$*,* $u_0 \in \mathrm{dom}(\Phi)$*. Then the problem* $\boldsymbol{P}(\Phi, \mathcal{B}, f, u_0)$ *has a unique solution, it belongs to the space* $L^\infty(0,T;V) \cap H^1(0,T;H)$*, while* $\Phi\big(u(\cdot)\big) \in C([0,T];H)$*, and satisfies estimates*

$$\max_{t\in[0,T]} |u(t)|^2 + \int_0^T \|u(t)\|^p \, dt + \int_0^T \Phi\big(u(t)\big)\, dt \leqslant C_1 \Big(|u_0|^2 + \int_0^T |f(t)|^2 \, dt\Big),$$

$$\operatorname*{ess\,sup}_{t\in[0,T]} \|u(t)\|^p + \max_{t\in[0,T]} \Phi\big(u(t)\big) + \int_0^T |u'(t)|^2 \, dt \leqslant C_2\Big(|u_0|^2 + \Phi(u_0) + \int_0^T |f(t)|^2 \, dt\Big),$$

where C_1, C_2 *are positive constants depending on* K*,* L*, and* T *only.*

Asymptotic solutions of linear optimal control problems

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Consider the problem described by the system of differential equations

$$\varepsilon^h B(t,\varepsilon)\frac{dx}{dt} = A(t,\varepsilon)x + C(t,\varepsilon)u,$$

where $A(t,\varepsilon)$ and $B(t,\varepsilon)$ are real $(n \times n)$ matrices, $C(t,\varepsilon)$ is a real $(n \times m)$ matrix, $x(t,\varepsilon)$ is an n-dimensional vector of state, $u(t,\varepsilon)$ is an m-dimensional control vector, $\varepsilon \in (0,\varepsilon_0]$ is a small parameter, $\varepsilon_0 \ll 1$; $h \in N$, and $t \in [0;T]$.

The problem is to find a control $u(t,\varepsilon)$ under the action of which the system passes from the state $x(0,\varepsilon) = x_1(\varepsilon)$ into the state $x(T,\varepsilon) = x_2(\varepsilon)$ for a finite time interval T by minimizing the functional

$$J = \frac{1}{2\varepsilon^h}\int_0^T (D(t,\varepsilon)u,u)\,dt \to \min_u,$$

where $D(t,\varepsilon)$ is a symmetric positive-definite matrix of the m-th order.

Thus, we can apply the Pontryagin method [1] to the solution of the problem of optimal control. Using the theory of asymptotic integration of singularly perturbed systems of differential equations with degenerations of A.M. Samoylenko, M.I. Shkil', V.P. Yakovets' [2], the method of construction of asymptotic solutions of the given optimal control problem is suggested. This method is based on the idea of reducing of given pairpoint boundary-value problem.

The algorithm of construction of the asymptotics in cases of simple and multiple spectrum of the boundary bundle of matrixes is worked out. The existence and uniqueness conditions for the solution of this optimal control problem in each case have been found.

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The mean value theorems on the regular polygon

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Let $n \in \mathbb{N}$, $n \geq 3$. There is well-known result in theory of harmonic functions, that was proved independently in works of Kakutani and Nagumo [1], Walsh [2] and Privalov [3]: a function $f \in C(\mathbb{C})$ is a harmonic polynomial of order $\leq n-1$ if and only if the mean value of the function f taken over vertices of any regular n-gon equals to the value of this function at its center.

Let $B_R := \{z \in \mathbb{C} : |z| < R\}$, $m, n \in \mathbb{N}$, $s \in \mathbb{N}_0$, $n \geq 3$, $s < m < n+1$, $d_n := 2(5+4\cos\frac{\pi}{n})^{-1/2}$ for odd n, $d_n := 2(4+5\cos^2\frac{\pi}{n})^{-1/2}$ for even n. Denote by $E(n,m,s)$ the set of all pairs of integer nonnegative numbers (k,l), such that the following conditions hold: $k < m-s$ or $l < m$; $k < n+s$; $l < n-s$.

Theorem. *Let $R > 0$, $f \in C^{2m-s-2}(B_R)$, $r \in (0,\ d_nR)$. Then the following assertions are equivalent:*
1) for all $z \in B_R$ and $\alpha \in [0,\ 2\pi)$ such that $\{z + re^{i\alpha+i\frac{2\pi\nu}{n}}\}_{\nu=0}^{n-1} \subset B_R$ we have the equality

$$\sum_{p=s}^{m-1} \frac{nr^{2p}}{(p-s)!p!} \partial^{p-s}\bar{\partial}^p f(z) = \sum_{\nu=0}^{n-1} (re^{i\alpha+i\frac{2\pi\nu}{n}})^s f(z + re^{i\alpha+i\frac{2\pi\nu}{n}});$$

2) the function f is represented in the form

$$f(z) = \sum_{(k,l)\in E(n,m,s)} c_{k,l} z^k \bar{z}^l, \quad c_{k,l} \in \mathbb{C}.$$

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Clarification of the Lakshmikantham inequality

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Significant contribution to the development of the theory of integral inequalities was made by the famous American mathematician V. Lakshmikantham. Let us consider one of his results [3].

Consider the differential equation:

$$u' = \lambda(t)g(u) + \sigma(t), \qquad u(t_0) = u_0 \geqslant 0, \tag{1}$$

where $\lambda \in C[I, \mathbb{R}]$, $g \in C[\mathbb{R}^+, \mathbb{R}^+]$, $g(0) = 0$ for $u > 0$, $\sigma \in C[I, \mathbb{R}]$ and $I = [t_0, T_0]$. Assuming that $\lambda(t) \geqslant 0$, $\sigma(t) \geqslant 0$ and $g(u)$ is a non-decreasing function, we obtain the inequality for the function $u(t)$:

$$u(t) \leqslant G^{-1}\left(\int_{t_0}^{t} \lambda(s)ds + G\left(u_0 + \int_{t_0}^{t} \sigma(s)ds\right)\right), \qquad t \geqslant t_0.$$

In this paper, the clarification of this inequality is proposed.

Let us formulate the main result of this work.

Consider the differential equation (1) and evaluate the function $u(t)$ from above. Let us denote $G(u) = \int_{u_0}^{u} \frac{ds}{g(s)}$, $\beta(t) = \dfrac{\sigma(t)dt}{g\left(u_0 + g(u_0)\int_{t_0}^{t} \lambda(\tau)d\tau\right)}$.

Theorem. *Let $g(u)$ be a differential function and $g'(u) \geqslant 0$, then function $u(t)$ satisfies the assessment:*

$$u(t) \leqslant G^{-1}\left(\int_{t_0}^{t} \lambda(\tau)d\tau + G(u_0) + \int_{t_0}^{t} \beta(\tau)d\tau\right), \qquad t \geqslant t_0.$$

By introducing additional assumptions about the nonlinear functions included in the right side of the comparison equation, new estimates are obtained for solutions of the nonlinear one-dimensional comparison equation that specify the Lakshmikantham inequality.

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Conditions for the finite speed of propagation of non-negative solutions of the Bussinesque equation with nonhomogeneous density

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We examine problem in fluid mechanics. It deals with the filtration of an incompressible fluid (typically, water) through a porous stratum, the main problem in groundwater infiltration. The model was developed first by Boussinesq in 1903. [1]

For this model we have Boussinesq's equation

$$\rho \frac{\partial u}{\partial t} = \Delta u^2$$

We know that solution of equation (1) has the property of a finite speed of propagation of perturbations, when $\rho = const$.

We assume that $\rho = (1 + |x|)^{-l}$, $l > 0$, and consider the Cauchy problem

$$\rho \frac{\partial u}{\partial t} = \Delta u^2 \tag{1}$$

in $Q_T = \mathbb{R}^3 \times (0, T)$,

$$u(x, 0) = u_0(x), \quad x \in \mathbb{R}^3, \quad u_0(x) \geq 0 \quad \text{for a.e.} \quad x \in \mathbb{R}^3. \tag{2}$$

There $u = u(x, t), \quad x = (x_1, x_2, x_3), \quad |x| = (x_1^2 + x_2^2 + x_3^2)^{\frac{1}{2}}$.

In addition

$$\sup u_0 \subset B_{R_0} \equiv \{|x| < R_0\}, \quad \|u_0\|_{\infty, \mathbb{R}^3} < \infty.$$

We say that the solution of the equation (1) has a property of a finite speed of propagation of perturbations if from the condition $\sup(\cdot, t) < \infty$ at some point in time $t_0 \geq 0$ it follows that this property is preserved for all moment times $t \geq t_0$. [2]

Let us formulate the main result of this work.

Theorem. *Let $u(x,t)$ – a solution of the problem (1), (2) in Q_T. If $0 < l < \frac{4}{3}$, then $u(x,t)$ has a property of a finite speed of propagation of perturbations.*

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Local boundedness and continuity of solutions to high-order quasilinear elliptic equations via Wolff potentials

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Let Ω be a bounded open set of $\mathbb{R}^n$, $n \geqslant 3$. We consider quasilinear $2m$-order ($m \geqslant 2$) partial differential equations in the divergent form:

$$\sum_{|\alpha|\leqslant m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, u, D^1 u, \dots, D^m u) = f(x), \quad x = (x_1, \dots, x_n) \in \Omega, \tag{1}$$

where $f \in L^1(\Omega)$, $\alpha = (\alpha_1, \dots, \alpha_n)$ is an n-dimensional multi-index with nonnegative integer components α_i, $i = 1, \dots, n$, $|\alpha| = \alpha_1 + \dots + \alpha_n$, $D^\alpha = \partial^{|\alpha|}/\partial x_1^{\alpha_1} \cdots \partial x_n^{\alpha_n}$ and $D^k u = \{D^\alpha u : |\alpha| = k\}$ for every $k = 0, 1, \dots, m$. We denote by Λ_m the set of all n-dimensional multi-indices α such that $|\alpha| \leqslant m$, and by $|\Lambda_m|$ the number of elements of the set Λ_m.

We make the following hypotheses on the coefficients $\{A_\alpha\}_{\alpha\in\Lambda_m}$ of Eq. (1).

(H1) For every $\alpha \in \Lambda_m$, $A_\alpha : \Omega \times \mathbb{R}^{|\Lambda_m|} \to \mathbb{R}$ is a Carathéodory function, i.e. for any $\xi \in \mathbb{R}^{|\Lambda_m|}$, the function $A_\alpha(\cdot, \xi)$ is measurable on Ω, and for almost every $x \in \Omega$, the function $A_\alpha(x, \cdot)$ is continuous in $\mathbb{R}^{|\Lambda_m|}$.

(H2) There exist the constants $K > 0$ and $p > 1$ such that for almost every $x \in \Omega$ and any $\xi \in \mathbb{R}^{|\Lambda_m|}$ the following inequalities hold:

$$\sum_{|\alpha| \leqslant m} |A_\alpha(x, \xi)| \leqslant K \sum_{1 \leqslant |\alpha| \leqslant m} |\xi_\alpha|^{p-1}, \quad \sum_{|\alpha|=m} A_\alpha(x, \xi)\xi_\alpha \geqslant K^{-1} \sum_{|\alpha|=m} |\xi_\alpha|^p.$$

To define solutions of Eq. (1), we use the Sobolev space $W^{m,p}(\Omega) = \{u : D^\alpha u \in L^p(\Omega), |\alpha| \leqslant m\}$, where $L^p(\Omega)$ denotes the usual Lebesgue space and $D^\alpha u$ is the α^{th} weak derivative of u (see, e.g., [2]). By definition, a generalized solution of Eq. (1) is a function $u \in W^{m,p}(\Omega)$ such that for every function $v \in C_0^\infty(\Omega)$,

$$\sum_{|\alpha| \leqslant m} \int_\Omega A_\alpha(x, u, D^1 u, \ldots, D^m u) D^\alpha v \, dx = \int_\Omega f v \, dx.$$

We denote by $B_r(y) := \{x \in \mathbb{R}^n : |x - y| < r\}$ the open ball with center y and radius r. We also use the following notion for the *Wolff potential* of the function f in the ball $B_\rho(y)$ (see [1] for more information):

$$\mathbf{W}^f_{a,p}(y; \rho) = \int_0^\rho \left(\frac{1}{r^{n-ap}} \int_{B_r(y)} |f(x)| \, dx \right)^{1/(p-1)} \frac{dr}{r}, \quad n \geqslant ap.$$

We extend the function f by zero on $\mathbb{R}^n \setminus \Omega$. For every $R > 0$, we set $\mathbf{W}^f_{m,p}(R) = \sup_{x \in \Omega} \mathbf{W}^f_{m,p}(x; R)$.

The following two theorems are the main results of our report.

Theorem 1. *Let $n = mp$, $u \in W^{m,p}(\Omega)$ be a generalized solution of Eq.* (1) *under assumptions* (H1) *and* (H2). *Let x_0 be a Lebesgue point of the function u, and let $B_{4R}(x_0) \subset \Omega$, $R \leqslant 1$. Then there exists a positive constant C_1 depending only on n, m and K such that*

$$|u(x_0)| \leqslant C_1 \left(R^{-n} \int_{B_R(x_0)} |u|^p \, dx \right)^{1/p} + C_1 \sum_{|\alpha|=1}^{m-1} \left(\int_{B_R(x_0)} |D^\alpha u|^{n/|\alpha|} \, dx \right)^{|\alpha|/n} + C_1 \mathbf{W}^f_{m,p}(x_0; 2R). \tag{2}$$

Moreover, if $\sup_{x \in B_R(x_0)} \mathbf{W}^f_{m,p}(x; 2R) < +\infty$ then $u \in L^\infty(B_R(x_0))$, and the following inequality holds:

$$\operatorname{ess\,sup}_{B_{R/2}(x_0)} |u| \leqslant C_2 \left(R^{-n} \int_{B_R(x_0)} |u|^p \, dx \right)^{1/p} + C_2 \sup_{x \in B_R(x_0)} \mathbf{W}^f_{m,p}(x; 2R),$$

where C_2 is a positive constant depending only on n, m and K.

Theorem 2. *Let $n = mp$, and let $u \in W^{m,p}(\Omega)$ be a generalized solution of Eq.* (1) *under hypotheses* (H1) *and* (H2). *If we assume that $\lim_{R \to 0} \mathbf{W}^f_{m,p}(R) = 0$ then u is locally continuous in Ω.*

Remark. To prove Theorem 1, we use an analogue of the Kilpeläinen-Malý method proposed in [3] for second-order equations ($m = 1$, $n \geqslant p > 1$), as well as modified test functions inspired by paper [5]. In turn, the proof of Theorem 2 is based on obtaining an estimate of the form (2) for some auxiliary function $V(u)$ suggested by [4].

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Boundary values with singular peaking for quasilinear heat equations

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The following problem for a quasilinear heat equation is considered:

$$
\begin{aligned}
&(|u|^{q-1}u)_t - \Delta_p(u) = 0, \quad (t,x) \in [0,T) \times \Omega, \quad p \geqslant q > 0,\\
&u(0,x) = u_0 \ \text{ in } \Omega, \quad u_0 \in L^{q+1}(\Omega),\\
&u(t,x)\Big|_{\partial\Omega} = f(t,x),
\end{aligned}
$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega \in C^2$. Boundary regime is defined by the function f which is from corresponding Sobolev space and has a singular peaking for the finite time interval, namely,

$$f(t,x) \to \infty \quad \text{as} \quad t \to T, \quad T < \infty.$$

In the papers [1] and [2] the asymptotic properties of weak (energy) solutions of the problem under consideration were studied. Accurate estimates of solution profiles were obtained. The dependence on the nature of the peaking of the boundary regime f was also studied.

The problem is investigated using the method of energy estimates. This method was announced and developed by A. E. Shishkov, A. G. Schelkov and V. A. Galaktionov in 1999–2006. It was used to obtain conditions of localization depending on the boundary regime f. In the mentioned papers, method of energy estimates was improved for studying the asymptotic behavior of solutions.

The obtained results were applied for studying equations with absorption potential:

$$(|u|^{q-1}u)_t - \Delta_p(u) = -b(t,x)|u|^{\lambda-1}u, \qquad (t,x) \in [0,T) \times \Omega, \quad \lambda > p \geqslant q > 0.$$

The absorption potential $b(t,x)$ is a continuous on $[0,T] \times \overline{\Omega}$ function which satisfies the following degenerate conditions:

$$b(t,x) > 0 \quad \text{в } [0,T) \times \overline{\Omega}, \qquad b(t,x) = 0 \quad \text{на } \{T\} \times \Omega,$$

In the paper [3] the asymptotic properties of weak solutions of the equation were studied under particular conditions on the character of degeneration of absorption potential. The precise estimates of solution profiles were obtained.

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